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[62/A22] SEAT No._____

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SARDAR PATEL UNIVERSITY

M. Sc. (Semester IV) Examination

Date: 08-11-2017, Wednesday **Time: 10.00 To 01.00 p.m.**

Subject: MATHEMATICS **Paper No. PS04EMTH29 – (Graph Theory – II)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) A shortest path between two vertices in a graph can be obtained using
(a) Kruscal algorithm (b) BFS algorithm
(c) Dijkstra's algorithm (d) none of these

(2) If all the digits in the Pruffer code are same, then the graph is
(a) Cycle graph (b) Path graph (c) Star graph (d) $K_{n,n}$ $(n > 1)$

(3) The number of spanning trees in C_n is
(a) n (b) n! (c) 1 (d) none of these

(4) In a network, if s is source and t is sink, then
(a) $d^+(s) = 0 = d^-(t)$ (b) $d^+(s) > 0, d^-(t) > 0$
(c) $d^+(s) = 0, d^-(t) > 0$ (d) $d^+(s) > 0, d^-(t) = 0$

(5) Let A be a matrix with spectrum $\{-1, -2, 2, 3\}$. Then Trace(A) =
(a) 12 (b) – 12 (c) –2 (d) 2

(6) Let G be a graph with $\chi(G) = 5$. Then $\lambda_{max}(G)$
(a) $= 4$ (b) ≤ 4 (c) ≥ 4 (d) none of these

(7) The Ramsey number R(3, 3) is
(a) 3 (b) 6 (c) 9 (d) none of these

(8) If $E = \{a, b, c\}$ with $M = \{\{a\}, \{b\}, \{a,b\}\}$ as hereditary system, then C_M =
(a) {c} (b) {{c}, {b,c}} (c) {{c}, {a,c}} (d) {{c}, {a,b,c}}

2. Attempt any SEVEN: [14]

(a) How many trees are there with degree sequence (2,1,1,2,2)?

(b) State Matrix-tree theorem.

(c) If f is a flow on a network N = (V, A), then find f({s}, V) and f({t}, V).

(d) Prove or disprove: Length of minimum weighted path from vertex u to v is d(u, v).

(e) Prove: If G is k regular graph, then k is an eigen value of G.

(f) Prove: $\lambda_{max}(G) \leq \Delta(G)$.

(g) Define u-v separating set and give one example of it.

(h) Prove: $R(p, 2) = p$, if $p \geq 2$.

(i) Prove: For $X \subset E$ and $e \in E$, $r(X + e) \leq r(X) + 1$.

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3. (a) Find $\tau(G)$ for $G = K_{2,3}$. [6]

(b) Construct a tree with Pruffer code (12321). [6]

OR

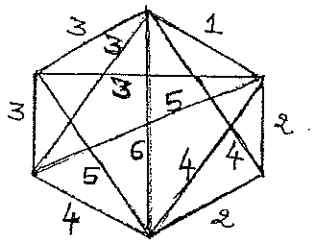
(b) Show that if a tree T with m edges has graceful labeling, then K_{2m+1} can be decomposed into (2m + 1) copies of T. [6]

4. (a) Let f be a flow on a network N = (V, A) with value d. Prove that, if $A(X, \overline{X})$ is a cut in N, then $d = f(X, \overline{X}) - f(\overline{X}, X)$. [6]

(b) Define a flow and value of a flow in a network and give one example of a flow in a network. [6]

OR

(b) Using Kruscal's algorithm, find a shortest spanning tree for the graph below: [6]



5. (a) Find $sp(C_4)$. [6]

(b) Let G be a bipartite graph. Prove that if λ is an eigenvalue of G with multiplicity m, then $-\lambda$ is also an eigenvalue of G with m multiplicity. [6]

OR

(b) Prove: The diameter of G is less than the number of distinct eigen values of G. [6]

6. (a) Prove: $R(p, q) \geq (p-1)(q-1) + 1$. [6]

(b) Prove (ANY ONE): In a hereditary system, [6]

(i) Sub modularity property (R) $\Rightarrow$ Weak elimination property (C).

(ii) Uniformity property (U) $\Rightarrow$ Base Exchange property (B).

x-x-x-x-x-x