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SEAT No.________

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SARDAR PATEL UNIVERSITY
M. Sc. (Semester III) Examination

Date: 11-11-2017, Saturday **Time: 10.00 To 01.00 p.m.**
Subject: MATHEMATICS **Paper No. PS03EMTH23 – (Graph Theory – II)**
Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The number of spanning trees in K_n $(n>1)$ is
(a) n (b) n^2 (c) n^{n-2} (d) n^n

(2) If all the digits in the Pruffer code are same, then the graph is
(a) Star graph (b) Path graph (c) Cycle graph (d) $K_{n,n}$ $(n>1)$

(3) The graph K_{11} can be decomposed into copies of
(a) $K_{1,6}$ (b) P_6 (c) P_5 (d) none of these

(4) If f is a flow on a network N = (V, A) with source s and sink t, then $f(\{s\},V) =$
(a) $f(V,\{s\})$ (b) $f(V,\{t\})$ (c) $f(\{t\}, V)$ (d) none of these

(5) Let A be a matrix with spectrum $\{-2, -1, 3, -3, 1\}$. Then det(A) =
(a) –2 (b) 9 (c) 18 (d) –18

(6) Let $G = K_{4,3}$. Then the non-zero eigen values for G is
(a) 2 (b) 3 (c) 4 (d) 12

(7) The Ramsey number R(p, q)
(a) $= \text{Min}\{p, q\}$ (b) $\leq \text{Min}\{p, q\}$ (c) $\geq \text{Max}\{p, q\}$ (d) $= \text{Max}\{p, q\}$

(8) If $E = \{1, 2, 3\}$ with $\mathcal{M} = \{\{3\}, \{2\}, \{2,3\}\}$ as hereditary system, then $r(\{1,2\}) =$
(a) 0 (b) 1 (c) 2 (d) 3

2. Attempt any SEVEN: [14]

(a) Construct a tree with Pruffer code (1234).

(b) Give one graceful labeling of P_6 with detail.

(c) Define a cut in a network and give one example of it.

(d) State Pigeonhole Principle.

(e) Prove: If G is k regular graph, then k is an eigen value of G.

(f) Prove: $sp(A^2) = \{\lambda^2 : \lambda \in sp(A)\}$.

(g) Define u-v separating set and give one example of it.

(h) Prove or disprove: R(3, 4) = 8.

(i) If $E = \mathbb{Z}$ with $\mathcal{M} = \{X \subset E; |X| < 7\}$ as hereditary system, then find $B_{\mathcal{M}}$ and $C_{\mathcal{M}}$.

(P.T.O)

3. (a) Prove: If $e \in E(G)$ is not a loop, then $\tau(G) = \tau(G-e) + \tau(G \bullet e)$. [6]

(b) Find $\tau(G)$ using Matrix-Tree theorem, for $G = K_{2,3}$. [6]

OR

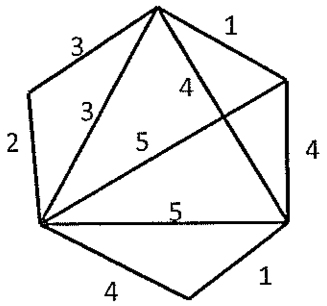
(b) How many trees are there with degree sequence (1,3,1,2,1)? Construct any one such tree. [6]

4. (a) Let f be a flow on a network N = (V, A) with value d. Prove that, if $A(X, \overline{X})$ is a cut in N, then $d = f(X, \overline{X}) - f(\overline{X}, X)$. [6]

(b) Define source, sink and flow in a network N and illustrate these concepts by giving one example of a network N with at least five vertices. [6]

OR

(b) Using Kruscal's algorithm, find a shortest spanning tree for the graph below: [6]



5. (a) Prove: For any graph G, $\chi(G) \leq 1 + \lambda_{\max}(G)$. [6]

(b) (i) Prove: If J is a linear combination of powers of A(G), then G is regular. [6]

(ii) Find sp(($K_{1,3}$).

OR

(b) (i) Prove: If G is bipartite graph, then eigen values of G occur in pair $(\lambda, -\lambda)$ $(\lambda \neq 0)$. [6]

(ii) Give an example of a simple graph G with $\delta(G) < \lambda_{\max}(G) = \Delta(G)$.

6. (a) Prove: $R(p, q) \leq R(p-1, q) + R(p, q-1)$, $\forall$ p, q > 2. [6]

(b) Prove: In a hereditary system, [6]
Uniformity property (U) $\Rightarrow$ Sub modularity property (R)

OR

(b) With usual notations, prove that $r(X) \leq r(X+e) \leq r(X) + 1$, for $X \subset E$ and $e \in E$. [6]

x-x-x-x-x-x