(85 & A-25) Seat No: ______

No of printed pages: 2

Sardar Patel University
Mathematics
M.Sc. Semester III
Wednesday, 19 October 2016
2.00 p.m. to 5.00 p.m.
PS03CMTH02 - Mathematical Methods I

Maximum Marks: 70

Q.1 Choose the correct option for each of the following. [8]

(1) Let $f(x) = \cos x + i \sin x$. Then the complex Fourier coefficient c_{-1} of f is

(a) 1 (b) i (c) $-i$ (d) 0

(2) Let f be a 2π- periodic function given by $f(x) = x - x^2$, $-\pi < x \leq \pi$. The Fourier series of f at π converges to

(a) $-\pi^2$ (b) π^2 (c) $-\pi$ (d) π

(3) If $f \in L^1(\mathbb{R})$ and $F[f](s) = \frac{\sin 4s}{3s}$, $s \neq 0$, then $F[f](0)$ is equal to

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) 1 (d) 0

(4) Let f be an even integrable function. Which of the following is not true?

(a) $F[f] = F^{-1}[f]$ (b) $F_c[f] = F[f]$ (c) $F[f] = -iF_s[f]$ (d) $F[F[f]] = F[f]$

(5) $L[e^{at} \sinh bt](t) =$

(a) $\frac{a}{(s-a)^2+b^2}$ (b) $\frac{b}{(s-a)^2+b^2}$ (c) $\frac{a}{(s-a)^2-b^2}$ (d) $\frac{b}{(s-a)^2-b^2}$

(6) $L^{-1}\left[\frac{L[f](s)}{s}\right](t) =$

(a) $\int_t^\infty L[f](u)du$ (b) $\int_0^t L[f](u)du$ (c) $\int_t^\infty f(u)du$ (d) $\int_0^t f(u)du$

(7) The Z- transform of $(\frac{1}{n!})_{n\geq 0}$ is

(a) e^z (b) e^{-z} (c) $e^{\frac{1}{z}}$ (d) $e^{-\frac{1}{z}}$

(8) The value of $\frac{d^{11}}{dx^{11}}[e^{x^2} \frac{d^7}{dx^7}(e^{-x^2})]$ at $x = 0$ is

(a) 7 (b) 11 (c) $\frac{7}{11}$ (d) none of these

Q.2 Attempt any *Seven*. [14]

(a) Compute the half range Fourier cosine series of $f(x) = 1$, $0 < x < \pi$.

(b) Express the complex Fourier coefficient c_n of a 2π- periodic function f in terms of its Fourier coefficients a_n and b_n.

(c) Evaluate $\int_0^\infty \frac{\sin^2 x}{x^2} dx$ using Fourier transform methods.

(d) Let $u(x,t)$ be a function of two variables such that both $u(x,t)$ and $u_x(x,t)$ tend to 0 as $x \to \infty$. In usual notations show that $F_s[u_{xx}](s) = -s^2 F_s[u](s) + \sqrt{\frac{2}{\pi}} s u(0,t)$.

(P.T.O.)

(e) Evaluate the Laplace transform of t^2e^{-4t}.
(f) Find the inverse Laplace transform of $\frac{e^{-\pi s}}{s^2+1}$.
(g) Compute $H_3(1)$.
(h) Compute the inverse Z- transform of $\frac{z}{(z-2)(z-3)}$.
(i) State Gram-Schmidt Orthonormalization Theorem.

Q.3
(a) Compute the Fourier series of 2- periodic function $f(x) = x - x^2$, $-1 < x \le 1$. Hence evaluate the sums of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^4}$ [6]
(b) Compute the half range Fourier cosine series of the function $f(x) = \frac{2x}{\pi}$ if $0 < x < \frac{\pi}{2}$ and $f(x) = \frac{2}{\pi}(\pi - x)$ if $\frac{\pi}{2} \le x < \pi$. Hence evaluate $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. [6]

OR

(b) Solve $u_t = c^2u_{xx}$, $0 < x < 100$, $t > 0$ subject to $u(0,t) = u(100,t) = 0$, $t \ge 0$, $u(x,0) = x$ for $0 \le x \le 50$ and $u(x,0) = 100 - x$ for $50 \le x \le 100$. (you may take both functions equal to $-k^2$ while applying separation of variables) [6]

Q.4
(c) Solve $u_{tt} = c^2u_{xx}$, $x,t > 0$ subject to $u(x,0) = f(x)$, $u_t(x,0) = g(x)$ for all $x > 0$ and $u(0,t) = 0$ for all $t > 0$ using Fourier transform methods. [6]
(d) Compute the Fourier transform of $\chi_{[-1,1]}$. Hence evaluate the integrals $\int_0^{\infty} \frac{\sin x \cos 2x}{x} dx$ and $\int_0^{\infty} \frac{\sin x}{x} dx$. [6]

OR

(d) (a) If $f, g \in L^1(\mathbb{R})$, then show that $f \star g \in L^1(\mathbb{R})$. [3]
(b) Let $\alpha > 0$ and $\alpha \ne 1$. Solve $y'' - y = e^{-\alpha|x|}$ subject to $y(x) \to 0$, $y'(x) \to 0$ as $|x| \to \infty$. [3]

Q.5
(e) Use Laplace transform methods to solve $u_{tt} = c^2u_{xx}$, $x > 0, t > 0$ subject to $u(0,t) = A\sin wt$, $u_x(0,t) = 0$ for all t and $u(x,0) = u_t(x,0) = 0$ for all x. [6]
(f) Compute the inverse Laplace transforms of the function $\frac{s}{s^4+s^2+1}$ and $\frac{e^{-3s}}{(s-1)^2+2}$. [6]

OR

(f) Solve $y'' + 2y' + 5y = e^t \sin t$ subject to $y(0) = 3$ and $y'(0) = 1$.

Q.6
(g) Orthonormalize the set $\{1, x, x^2\}$ over $[-1, 1]$ and show that [6]

$$\text{sp}\{1, x, x^2, \ldots\} = \text{sp}\left\{\sqrt{\frac{2n+1}{2}}P_n(x) : n \in \mathbb{N} \cup \{0\}\right\}.$$

(h) (a) Solve $y_{n+2} - 6y_{n+1} + 5y_n = 0$, $n \ge 0$, subject to $y_0 = 1$ and $y_1 = -2$. [3]
(b) Find the Green's function for $y''(x) + 9y(x) = f(x)$ subject to $y(0) = 0$, $y'(\pi) = 0$ and hence find the solution of the above equation where $f(x) = x\cos x$. [3]

OR

(h) (a) Find a polynomial of degree 2 so that $\int_{-1}^{1} |\sin x - p(x)|^2 dx$ is minimum. [3]
(b) Show that $H_{n+1}(x) - 2xH_n(x) + 2nH_{n-1}(x) = 0$ for all $n \ge 1$ and hence show that $H_n(x)$ satisfies $y'' - 2xy' + 2ny = 0$. [3]

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2