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Sardar Patel University

Mathematics

M.Sc. Semester III

Saturday, 1<sup>st</sup> December 2012

2.30 p.m. to 5.30 p.m.

PS03QMTM03 - Mathematical Methods I

Maximum Marks: 70

Q.1 Choose the correct option for each of the following.

(1) Let  $f = \chi_Q$  and  $g = \chi_{R \setminus Q}$  be 2-periodic functions. Then the Fourier series of  $f + g$  and  $fg$  at 1 will converge, respectively, to [8]

- (i) 1 and 0      (ii) 0 and 1      (iii) 1 and 1      (iv) 0 and 0

(2) Which of the following functions does not satisfy the Dirichlet condition of (0, 1)?

- (a)  $\sin x$       (b)  $x \sin x$       (c)  $\sin(\frac{1}{x})$       (d)  $x^2 \cos x$

(3) Let  $f \in L^1(\mathbb{R})$  be an odd function. Which of the following is true?

- (a)  $F[f] = F_c[f]$       (b)  $F[f] = -F_c[f]$       (c)  $F[f] = iF_s[f]$       (d)  $F[f] = -iF_s[f]$

(4) Let  $f \in L^1(\mathbb{R})$ . Which of the following conditions ensures that  $f = F^{-1}[F[f]]$  a.e.?

- (a)  $f$  is continuous      (c)  $F[f] \in L^1(\mathbb{R})$   
 (b)  $f \in C_0(\mathbb{R})$       (d) none of above

(5) If  $\bar{f}$  is the Laplace transform of  $f$ , then  $L\left[\frac{f(t)}{t}\right](s) =$ 

- (a)  $\int_0^\infty \bar{f}'(u) du$       (b)  $\int_s^\infty \bar{f}'(u) du$       (c)  $\int_0^s \bar{f}(u) du$       (d)  $\int_s^\infty \bar{f}(u) du$

(6) The inverse Laplace transform of  $\frac{e^{xt}}{s^2+1}$  is

- (a)  $\sin(t+\pi)H(t+\pi)$       (c)  $\sin(t+\pi)H(t-\pi)$   
 (b)  $\sin(t-\pi)H(t-\pi)$       (d)  $\sin(t-\pi)H(t+\pi)$

(7) Which of the following is the Hermite polynomial of degree  $n$ ?

- (a)  $e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$       (b)  $e^{-x^2} \frac{d^n}{dx^n}(e^{x^2})$       (c)  $e^{-x^2} \frac{d^n}{dx^n}(e^{-x^2})$       (d) None of these

(8) The domain of the convergence of the Z-transform of  $(\frac{1}{3^n})_{n \geq 0}$  is

- (a)  $|z| > 3$       (b)  $|z| < 3$       (c)  $|z| > \frac{1}{3}$       (d)  $|z| < \frac{1}{3}$

Q.2 Attempt any Seven.

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- (a) Compute the half range Fourier sine series of  $f(x) = e^x$ ,  $0 < x < \pi$ .
- (b) Find the complex Fourier coefficient  $c_0$  of the function  $f(x) = \cos^4 x + \sin^4 x$ .
- (c) Evaluate  $\int_0^\infty \frac{\sin x}{x(x^2+4)} dx$  using Fourier transform methods.
- (d) Let  $f, g \in L^1(\mathbb{R})$ . Show that  $F[f * g] = F[f]F[g]$ .
- (e) Compute the Laplace transform of  $e^{4t} t \sin t$ .
- (f) Compute the inverse Laplace transform of  $\frac{s^2}{(s^2+1)(s^2+9)}$ .
- (g) State the Gram-Schmidt orthonormalization theorem.
- (h) Show that any orthogonal set, not containing zero, is linearly independent.
- (i) Compute the Z-transform of the sequence  $(n^2)_{n \geq 0}$ .

Q.3

- (a) Solve  $\nabla^2 u = 0$ ,  $r > a$ ,  $0 \leq \theta \leq 2\pi$ , subject to the conditions  $u(a, \theta) = f(\theta)$ ,  $0 \leq \theta \leq 2\pi$ , where  $f$  is a continuous function, and  $u$  is bounded in a variable  $r$ . (you may assume that  $r^{-n}(a_n \cos n\theta + b_n \sin n\theta)$ , for each  $n \in \mathbb{N}$ , and a constant function are solutions of  $\nabla^2 u = 0$ .)
- (b) Let  $f(x) = |x|$ ,  $-\frac{\pi}{2} < x \leq \frac{\pi}{2}$ . Use the Parseval's formula to evaluate the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ .

OR

- (b) Let  $f(x) = x(\pi - x)$ ,  $0 < x < \pi$ . Use the half range Fourier sine or cosine series to evaluate the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{n^4}$ .

Q.4

- (c) Use Fourier transform methods to solve  $u_{xx} + u_{yy} = 0$  ( $x \in \mathbb{R}, y > 0$ ) subject to  $u(x, 0) = f(x)$  ( $x \in \mathbb{R}$ ),  $u$  is bounded as  $y \rightarrow \infty$  and both  $u$  and  $u_x$  tend to 0 as  $|x| \rightarrow \infty$ .
- (d) Define convolution of  $f, g \in L^1(\mathbb{R})$ . If  $f(x) = \frac{1}{1+x^2}$ , then compute  $f * f$  stating all the results you use.

OR

- (d) Compute the Fourier transform of  $e^{-\frac{x^2}{4}}$ . Hence compute the Fourier transform of  $x^2 e^{-\frac{x^2}{4}}$ .

Q.5

- (e) Use Laplace transform methods to solve  $u_{xx} = \frac{1}{2}u_{tt} + k$ ,  $(0 < x < \ell, t > 0)$ , where  $a$  and  $k$  are constants, subject to the conditions  $u(0, t) = u(\ell, t) = 0$  ( $t > 0$ ) and  $u(x, 0) = u_t(x, 0) = 0$  ( $0 \leq x \leq \ell$ ).
- (f) Use Laplace transform methods to solve the simultaneous differential equations  $\frac{dy}{dt} + 3y - 2x = 0$ ,  $2x + \frac{dy}{dt} - y = 0$  subject to  $x(0) = 8$  and  $y(0) = 3$ .

OR

- (f) Use Laplace transform methods to solve  $ty'' + 2y' + ty = \sin t$  subject to  $y(0) = 1$ .

**Q.6**

- (g) Derive a formula to find  $n^{\text{th}}$  term of the Fibonacci sequence using Z-transform methods. [6]
- (h) (i) Find the Green's function for the boundary value problem  $y''(x) = f(x)$ ,  
 $y(0) = 0$  and  $y(1) = 0$ . [3]  
(ii) Orthonormalize  $\{1, x, x^2\}$  over  $(0, \infty)$  with respect to the weight function  $e^{-x}$ . [3]
- OR**
- (h) (i) Show that  $H_{n+1}(x) - 2xH_n(x) + 2nH_{n-1}(x) = 0$  for all  $n \in \mathbb{N}$ . [3]  
(ii) Find a polynomial  $p(x)$  of degree 2 so that  $\int_{-1}^1 (\cos x - p(x))^2 dx$  is minimum. [3]

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