(17 & A-7) Seat No: ______

No of printed pages: 2

Sardar Patel University
Mathematics
M.Sc. Semester II
Tuesday, 18 October 2016
10.00 a.m. to 1.00 p.m.
PS02CMTH01 - Real Analysis I

Maximum Marks: 70

Q.1 Fill in the blanks. [8]

(1) The Lebesgue measure of $[-1, 1)$ is ____

(a) 2 (b) 3 (c) 4 (d) none of these

(2) The Lebesgue integral of a nonnegative measurable function over $\mathbb{Q}$ is ____

(a) 1 (b) 2 (c) ∞ (d) none of these

(3) Let E be the set of irrationals in $[-1, 1]$. Then $\int_{[-1,1]} \chi_{E^c} =$ ____

(a) 1 (b) 0 (c) 2 (d) ∞

(4) The value of $\lim\limits_{n\to\infty} \int\limits_{[3,9]} \frac{nx}{1+nx} dx =$ ____

(a) 2 (b) 3 (c) 6 (d) none of these

(5) Fatou's Lemma can be obtained by making use of ____

(a) BCT (b) LDCT (c) MCT (d) none of these

(6) The outer measure m^* fails to have the property ____

(a) monotone
(b) m^* is countably subadditive
(c) translation invariant
(d) none of these

(7) Let $X = \{1, 2, 3\}$. Then the σ- algebra generated by $\{\{1\}\}$ contains ____ number of elements.

(a) 2
(b) 4
(c) 6
(d) none of these

(8) The false statement is ____

(a) f is measurable, $f = g$ a.e. imply g is measurable.
(b) f is continuous, $f = g$ a.e. mean g is continuous.
(c) f is integrable, $f = g$ a.e. imply g is integrable.
(d) none of these

(P.T.O.)

Q.2 Attempt any *Seven*. [14]
(a) Show that $P_a^b(f) + N_a^b(f) = T_a^b(f)$.
(b) Using a well-known result show that $m^*(\mathbb{Q}) = 0$.
(c) If f is increasing on $[a, b]$, then show that $f \in BV[a, b]$.
(d) If $|f|$ is integrable and f is measurable, then show that f is integrable.
(e) If f is measurable, then show that f^+ and f^- are measurable.
(f) Obtain BCT from LDCT.
(g) Show that a set having outer measure zero is measurable.
(h) If f and g are absolutely continuous on $[a, b]$, then show that $f + g$ is absolutely continuous on $[a, b]$.
(i) Suppose that $f = g$ a.e. on E. Then show that $\int_E f = \int_E g$.

Q.3
(a) Suppose f is integrable on $[a, b]$. Then show that the indefinite integral of f is a continuous function of bounded variation. [6]
(b) State and prove the Fundamental Theorem of Integral Calculus for an integrable function. [6]

OR

(b) Show that every absolutely continuous function is the indefinite integral of its derivative. [6]

Q.4
(c) Prove that without loss of generality any sequence of sets in an algebra can be considered to be a sequence of disjoint members. [6]
(d) Prove that outer measure of a finite closed interval is its length. [6]

OR

(d) If $\{f_n\}$ is a sequence of measurable functions, then show that $\inf_n f_n$ and $\sup_n f_n$ are measurable and hence show that limit of f_n is measurable. [6]

Q.5
(e) Show that $\int_E(\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$ if f and g are nonnegative measurable functions and $\alpha, \beta \geq 0$. [6]
(f) Prove that the Lebesgue integral of a nonnegative measurable simple function generates a measure. [6]

OR

(f) State and prove Bounded Convergence Theorem. Explain its meaning. [6]

Q.6
(g) Suppose $\{f_n\}$ is a sequence of measurable functions defined on a measurable set E and $f_n \to f$ a.e. on E. Then when does $\int_E f_n \to \int_E f$? Justify the answer. [6]
(h) State and prove MCT. Illustrate it by an example. [6]

OR

(h) Show that the Lebesgue integral is linear. [6]