Note: Figures to the right indicate full marks of the questions. (Total Marks: 70)

1   **Attempt all, write correct answers**   08

(i)   For an infinte interval, _____ measure should be used because both the _____ _
_____ measure and the _____ measure values are equal to ___
a) L-S, Lebesgue, Counting, ∞
b) L-S Measure, Lebesgue, Counting, 0
c) Lebesgue, L-S, Counting, ∞
d) Lebesgue, Counting, L-S, different

(ii)   *Lim $\int I(An) \, dP$ over Borel set B is _____following _____*
a) $P(A)$, MCT
b) $P(A \cap B)$, DCT
c) $P(A \cap B)$, MCT
d) $P(A \cap B)$, Fatous' theorem

(iii)   Lebesgue measure is a particular case of L-S measure for choice of Function
a) $F(x) = x-2$, x in R
b) $F(x) = x$, x in R+
c) $F(x) = x$, x in N
d) None of the above

(iv)   If $A_k = [1-1/k, 1+1/k)$ then $U A_k$ for k≥n is
a) null set
b) (1, 1)
c) [1-1/n, 1+1/n)
d) [1, 1)

(v)   The field containing {1,2 ..., 100} has ___ elements
a) 1
b) cardinality of power set
c) 100
d) 2 raise to 100

(vi)   The sufficient condition for a sequence of independent random variables having
zero odd moments to hold strong law of large number is

a) Variance is finite   b) Mean is zero
c) Third moment is finite   d) Fourth moment is finite
e)

(vii)   The $< X_n >$ converges in probability to X then this imply

a) $< X_n >$ converges in r[th] mean   b) $< X_n >$ converges a.s.
c) $< X_n >$ converges in distribution   d) None of these

(viii) The sequence of standardized sum of iid Bernouli random variables converges to a
      a) Binomial r.v.   b) Normal r.v.
      c) Degenerate r.v.   d) Poisson r.v.

2   **Attempt ANY 7, each carries 2 marks**   **14**

(a)   Prove that: If $A_n \rightarrow A$ then $A_n^c \rightarrow A^c$.

(b)   Answer, what is $\lambda(0, 1/n]=$ and $\lambda(N)=$ and why?

(c)   Show that indicator function of set A is measurable if and only if A is a measurable set.

(d)   Prove in usual notations that $\int_\Omega s+t\, d\mu = \int_\Omega s\, d\mu + \int_\Omega t\, d\mu$

(e)   Using Jenson's inequality prove that $E^{1/r}|X|^r \leq E^{1/s}|X|^s$ for $0 < r < s$.

(f)   State and prove Borel-Cantelli lemma.

(g)   Let X_n be a sequence of random variables defined by $P(X_n = 0) = 1-1/n^r$ and $P(X_n = n) = 1/n^r$ $r>0$, $n \geq 1$. Verify that $X_n \rightarrow 0$ in probability but not in rth mean.

(h)   Show that $F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1-x+x^2/2 & 1 \leq x \leq 2 \\ 1 & 2 \leq x \end{cases}$ is continuous distribution function.

(i)   Let $\{X_n\}$ be sequence of independent random variables with common uniform distribution over (0, 1). For $A_n = (X \leq 1/n)$, find the probability $P(\limsup A_n)$.

(j)   If $X_1, X_2 ..., X_n$ are iid bernouli random variables find the characteristic function of $Y = \sum_1^n X_i$.

3(a)   Define semi-field, field and sigma field giving example considering $\Omega = (0,1]$.   **06**
          What are the interrelationships among these classes?

3(b)   Define counting measure, Lebesgue measure and Lebesgue-Steiltjes measure.   **06**
          Show that each measure function is sigma finite and can be extended to a probability measure.

                                  OR
3(b)   Show that probability measure is continuous.

4(a)   Define Borel measurable function. Show that if f is non-negative measurable   **06**
          function then $f^2 + \alpha$ is also non-negative measurable, α is a real constant.

4(b)   Show that increasing sequence of non-negative simple functions converges to a   **06**
          non-negative measurable function.

                                      OR
4(b)   State and prove monotone convergence theorem.

5(a)   Prove that, if $y=g(x)$ is differentiable for all x, and either $g'(x) >0$ or <0 for all x **06**
          and if X is continuous then $Y=g(X)$ is continuous. Also obtain the density function
          of Y. If the pdf of random variable X is $f(x) = \alpha\beta x^{\beta-1} \exp(-\alpha x^\beta)$ for $x \geq 0$ and 0
          otherwise, then find pdf of $y = x^\beta$.

5(b)   State and prove Basic inequality.   **06**

5(b) If X is a random variable taking values $-t$, 0, t with probabilities p, 1-2p, p respectively, show that Markov's inequality attains equality.

6(a) Show that characteristic function is uniformly continuous and differentiable twice   06
if first and second moments are finite.

6(b) State and prove Kintchin's weak law of large numbers. Using this what can you   06
say about the sequence $<X_n>$ having pmf $P(X_n = n) = \frac{1}{2n} = P(X_n = -n)$ and

$P(X_n = 0) = 1 - \frac{1}{n}$. Does this sequence hold Weak law any way?

OR

6(b) State and prove Lindberg-Levy central limit theorem (CLT). State one application of this CLT.