

(A-2)

Seat No. _____

No of printed pages: 2

Sardar Patel University

M.Sc. Semester I Examination

2016

Friday, 21 October

10.00^{am} to 13.00 PM

Mathematics: PS01CMTH07

(Topology I).

Maximum Marks: 70

Q.1 Write the question number and correct option number only for each question.

[8]

(a) $\{(a, b) : a, b \in \mathbb{Z}, a < b\}$ is _____.(i) a base for a topology on \mathbb{Q} (iii) an open cover of \mathbb{R} (ii) a base for a topology on \mathbb{N}

(iv) family of disjoint intervals

(b) All topologies on _____ are compact.

(i) a finite set

(ii) an infinite set

(iii) \mathbb{Q} (iv) \mathbb{N}

(c) A compact metric space must be _____.

(i) countable

(ii) finite

(iii) complete

(iv) infinite

(d) \mathbb{R} with _____ topology is T_2 and separable.

(i) cocountable

(ii) discrete

(iii) usual

(iv) cofinite

(e) _____ with usual metric is not complete.

(i) \mathbb{Z} (ii) \mathbb{Q} (iii) $[0, 1]$ (iv) \mathbb{R} (f) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = -x$ is discontinuous if \mathbb{R} has _____ topology.

(i) discrete

(ii) cocountable

(iii) cofinite

(iv) lower limit

(g) Projections are _____.

(i) closed

(ii) open

(iii) one-one

(iv) homeomorphism

(h) A compact T_2 -space is

(i) discrete

(ii) T_3

(iii) separable

(iv) bounded

Q.2 Attempt any Seven. (Start a new page.)

[14]

(a) Prove that $\{\{-n, n\} : n \in \mathbb{Z}\}$ is a base for some topology on \mathbb{Z} .(b) Define *interior of a subset* of a topological space. Find the interior of $[0, 1]$ in \mathbb{R} with the usual topology.(c) Define *closure of a subset* of a topological space. Find closure of $(0, 1)$ in \mathbb{R} .

(d) Show that product of indiscrete topological spaces is an indiscrete topological space.

(e) Show that a constant function is always continuous.

(f) Show that $\{(0, n) : n \in \mathbb{N}\}$ has finite intersection property.(g) Define a *bounded* subset of a metric space and show that \mathbb{Q} is not bounded.(h) Define T_1 -space and show that \mathbb{R} is T_1 .(i) In usual notations prove that $\overline{(X \setminus A)} = X \setminus A^\circ$.

(P.T.O.)

[Contd...]

Q.3 (Start a new page.)

- (a) Show that intersection of two topologies on \mathbb{R} is again a topology on \mathbb{R} , but union of two topologies on \mathbb{R} need not be a topology on \mathbb{R} . [6]
- (b) Show that arbitrary union of closed sets need not be closed. Is a finite union of closed sets is closed? Justify. [6]

OR

- (b) Show that $A \subset \mathbb{R}$ is closed if and only if it contains its boundary. [6]

Q.4 (Start a new page.)

- (a) Define a *continuous function* and show that composition of two continuous functions is a continuous function. [6]
- (b) Let X be a complete metric space and $\{F_n : n \in \mathbb{N}\}$ be a family of closed subsets of X such that $F_{n+1} \subset F_n$ for all $n \in \mathbb{N}$. If $\text{diam}(F_n) \rightarrow 0$, then show that $\bigcap_{n=1}^{\infty} F_n$ is singleton. [6]

OR

- (b) Show that projections are continuous and open. [6]

Q.5 (Start a new page.)

- (a) Define a *compact* topological space and show that a closed subspace of a compact space is compact. [6]
- (b) Show that a sequentially compact metric space has Bolzano-Weierstrass Property. [6]

OR

- (b) Show that continuous image of a compact space is compact. [6]

Q.6 (Start a new page.)

- (a) Define a T_3 -space and show that a subspace of a T_3 -space is T_3 . [6]
- (b) Show that a metric space is T_4 . [6]

OR

- (b) Define a T_4 -space and show that a normal space is T_3 . [6]

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