(47)

Seat No.____

No of printed pages: 2

Sardar Patel University

M.Sc. Semester I Examination

2016

Friday, 21 October

10.00 am to 13.00 pm

Mathematics: PS01CMTH02

(Topology I)

Maximum Marks: 70

Q.1 Write the question number and correct **option number only** for each question. [8]

(*a*) If $\mathscr{B}$ is a base for a topology $\mathscr{T}$ on X, then ____.

(i) $\mathscr{B} \subset \mathscr{T}$ (ii) $\mathscr{B} = \mathscr{T}$ (iii) $\mathscr{T} \subset \mathscr{B}$ (iv) $X \in \mathscr{B}$

(*b*) ____ topology is the weakest topology on $\mathbb{R}$.

(i) cocountable (ii) usual (iii) indiscrete (iv) lower limit

(*c*) $\mathbb{R}$ with ____ topology is compact.

(i) cocountable (ii) usual (iii) indiscrete (iv) lower limit

(*d*) $\mathbb{R}$ with ____ topology disconnected.

(i) cocountable (ii) usual (iii) indiscrete (iv) lower limit

(*e*) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is discontinuous if $\mathbb{R}$ has ____ topology.

(i) cocountable (ii) usual (iii) indiscrete (iv) lower limit

(*f*) Complete metric space is ____.

(i) compact (ii) connected (iii) discrete (iv) of second category

(*g*) Projections are ____.

(i) closed (ii) open (iii) one-one (iv) homeomorphism

(*h*) A compact T_2-space is

(i) discrete (ii) T_3 (iii) connected (iv) bounded

Q.2 Attempt any **Seven**. **(Start a new page.)** [14]

(*a*) Prove that $\{(-n, n) : n \in \mathbb{N}\}$ is a base for some topology on $\mathbb{R}$.

(*b*) Find the boundary points of $\mathbb{N}$ in $\mathbb{R}$ with the usual topology.

(*c*) Show that $\mathbb{R}$ with discrete topology is T_1.

(*d*) Show that a finite product of discrete topological spaces is a discrete topological space.

(*e*) State one result ensuring the completeness of $[0, 1]$ with the usual topology.

(*f*) Show that $\{(0, r) : r > 0\}$ has finite intersection property.

(*g*) Define *totally bounded* metric space and show that $\mathbb{R}$ with usual metric is not totally bounded.

(*h*) Show that a finite set is compact with every topology on it.

(*i*) Show that a finite subset of $\mathbb{R}$ with the usual topology is disconnected.

Q.3 **(Start a new page.)**

(a) State and prove Pasting Lemma. [6]

(b) Show that every T_2-space is T_1 but the converse is not true. [6]

OR

(b) In $\mathbb{R}$ with the usual topology, find the limit points of (i) $\mathbb{Q}$, (ii) $\mathbb{N}$ and (iii) $\{1 + \frac{1}{n} : n \in \mathbb{N}\}$. [6]

Q.4 **(Start a new page.)**

(a) Let X be a complete metric space and $\{F_n : n \in \mathbb{N}\}$ be a family of closed subsets of X such that $F_{n+1} \subset F_n$ for all $n \in \mathbb{N}$. If $\text{diam}(F_n) \to 0$, then show that $\bigcap\limits_{n=1}^{\infty} F_n$ is singleton. [6]

(b) Define (i) a *continuous function*, (ii) a *uniformly continuous function* and prove that a continuous function on a metric space need not be uniformly continuous. [6]

OR

(b) For topological spaces $X_1, X_2, \ldots, X_n$, show that X_i is homeomorphic to a subspace of $\prod\limits_{i=1}^{n} X_i$ [6]

Q.5 **(Start a new page.)**

(a) Show that a topological space X is compact if and only if every family of closed subsets of X with FIP has a nonempty intersection. [6]

(b) Show that sequentially compact metric space X has Bolzano-Weierstrass Property. [6]

OR

(b) Show that a compact metric space is totally bounded but the converse is not true. [6]

Q.6 **(Start a new page.)**

(a) Let X be a topological space. Show that X is disconnected if and only if there is a nonempty proper clopen subset of X if and only if there is an continuous function from X onto $\{0, 1\}$. [6]

(b) Show that a compact T_2-space is regular. [6]

OR

(b) Let X be a topological space. Show that X is T_4 if and only if for every open set $V \subset X$ and a closed subset $F \subset V$, there exists an open set U in X such that $F \subset U \subset \overline{U} \subset V$. [6]

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2