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[160] SEAT No.________ No. of printed pages: 2

SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - I Examination

Tuesday, 07^{th} November, 2017

PS01CMTH02, Topology-I

Time: 02:00 p.m. to 05:00 p.m. Maximum marks: 70

Note: Figures to the right indicate full marks of the respective questions.
Assume standard notations wherever applicable.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) _____ topology on $\mathbb{R}$ is the smallest T_1-topology.
(i) Cofinite (ii) Usual (iii) Lower limit (iv) Discrete

(b) No subset of $\mathbb{R}$ with _____ topology has a limit point.
(i) cofinite (ii) usual (iii) lower limit (iv) discrete

(c) A polynomial of degree _____ defines a uniformly continuous function on $\mathbb{R}$.
(i) 1 (ii) 2 (iii) 3 (iv) 4

(d) Diameter of $\mathbb{R}$ with the metric $d(x, y) = \frac{|x-y|}{1+|x-y|}$, $(x, y \in \mathbb{R})$, is _____.
(i) 1 (ii) 2 (iii) 3 (iv) ∞

(e) _____ is a dense as well as a subset of first category in $\mathbb{R}$.
(i) $\mathbb{N}$ (ii) $\mathbb{Z}$ (iii) $\mathbb{Q}$ (iv) $\mathbb{R}$

(f) _____ subset of a metric space need not be closed.
(i) complete (ii) compact (iii) countable (iv) derived set of a subset

(g) _____ topology makes every set a compact topological space.
(i) usual (ii) cofinite (iii) discrete (iv) cocountable

(h) _____ topology on $\mathbb{R}$ is T_3 but not regular.
(i) usual (ii) lower limit (iii) discrete (iv) indiscrete

Q-2 Attempt *Any Seven* of the following: [14]

(a) Show that $\{(a, \infty) : a \in \mathbb{R}\}$ is a base for a topology on $\mathbb{R}$.

(b) Find the closure of $\{1, 2\}$ in cofinite topology on $\mathbb{R}$.

(c) Give a topology on $\{1, 2, 3, 4, 5, 6\}$ making it a T_2-space.

(d) Define the term *product topology*. Give an example of an open subset of $\mathbb{R} \times \mathbb{R}$.

(e) Define the term *metric on a set* and give an example of a metric space.

(f) Define the term *diameter of a subset of a metric space*. Find the diameter of $\{2 + \frac{1}{n} : n \in \mathbb{N}\} \cup \{3 - \frac{1}{n} : n \in \mathbb{N}\}$ in $\mathbb{R}$ with the usual metric.

(g) Define the term *open cover*. Give an open cover of $\mathbb{R}$ with the usual topology.

(h) Define the term *a separable topological space* and show that $\mathbb{R}$ with the usual topology is separable.

(i) Define and give an example of a *disconnected space*.

(P.T.O.)

1

Q-3 (j) Define the *usual and the lower limit topologies* on $\mathbb{R}$ and prove that the lower limit topology is finer than the usual topology. [6]

(k) Define *interior and closure of a subset* of a topological space. Show that a subset A of a topological space X is open if and only if $A = A^{\circ}$. [6]

OR

(k) State and prove Pasting Lemma. [6]

Q-4 (l) Show that the product topology is the weakest topology on the product space with respect to which the projections are continuous. [6]

(m) State and prove Cantor's Intersection Theorem [6]

OR

(m) Consider the product space $X = \prod_{i=1}^{n} X_i$. Then show that X is T_2 if and only if each X_i is T_2. [6]

Q-5 (n) Let $(X, \mathscr{T})$ be a topological space and $(Y, \mathscr{T}_Y)$ be its subspace. Show that Y is compact in Y if and only if Y is compact in X. [6]

(o) Show that a compact subset of a T_2-space is closed. [6]

OR

(o) Show that a compact metric space is totally bounded but the converse does not hold. [6]

Q-6 (p) Show that a topological space X is T_3 if and only if for every open set $G \subset X$ and a point $x \in G$, there exists an open set $H \subset X$ such that $x \in H \subset \overline{H} \subset G$. [6]

(q) Show that a metric space is T_4. [6]

OR

(q) Show that a continuous image of a connected space is connected. [6]

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2