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[98] SEAT No._____ NO. OF PRINTED PAGES: 2

SARDAR PATEL UNIVERSITY
M.Sc. (Semester-I) Examination

Tuesday 14/11/2017 **Time: 02:00 PM to 05:00 PM**

Subject: Mathematics **Course No.PS01EMTH02**

Mathematical Classical Mechanics

Note:
(1) All questions (including multiple choice questions) are to be answered in the answer book only.
(2) Numbers to the right indicate full marks of the respective question.

Q-1 Choose most appropriate answer from the options given. (08)

(1) A particle is moving on a cylinder, its degrees of freedom is_____.
(a) 0 (b) 2 (c) 4 (d) can not be determined

(2) The motion of a particle under gravity is _____ constraint.
(a) not a (b) a holonomic (c) a non-holonomic (d) conservative

(3) The condition for extremum of $J = \int_{x_1}^{x_2} f(y, x)dx$ is _____.
(a) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0$ (b) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial y} = 0$
(c) $\frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) - \frac{\partial f}{\partial \dot{y}} = 0$ (d) none of these

(4) If the Lagrangian L does not depend on q_j explicitly then _____ is conserved.
(a) p_j (b) h (c) $\dot{p}_j$ (d) L

(5) Which one of the following is correct?
(a) $\frac{\partial L}{\partial t} = \frac{dh}{dt}$ (b) $H = h$ (c) $\frac{dL}{dt} = \frac{dH}{dt}$ (d) none of these

(6) If all coordinates are non-cyclic then Routhian R = _____.
(a) H (b) L (c) $-H$ (d) 0

(7) Pick up the incorrect statement.
(a) A canonical transformation is non-invertible.
(b) Jacobian matrix for a canonical transformation is symplectic.
(c) Inverse of a canonical transformation is canonical.
(d) None of the above.

(8) $[q_1, p_2]$ is _____.
(a) a fundamental Lagrange bracket (b) a fundamental Poisson bracket
(c) a zero matrix (d) an undefined term

Q-2 Answer any **Seven.** (14)

(1) Define and give an example of a rheonomic constraint.
(2) Describe constraints in Atwood's machine.
(3) What are geodesics on a unit sphere?
(4) Define generalized momentum conjugate to a generalized coordinate.
(5) Explain the meaning of Legendre transformation in brief.
(6) State principle of least action.
(7) State the transformation generated by a function of type F_2.
(8) Define Poisson bracket.
(9) Evaluate $\{p_1, q_1 + p_2\}$, notations being usual.

(P.T.O.)

Q-3

(a) State Lagrange's equations of motion in general for and derive the form in the case of velocity dependent potential. (06)

(b) Giving all details obtain expression of Lagrangian for spherical pendulum. (06)

OR

(b) Express kinetic energy of system in terms of generalized coordinates and velocities.

Q-4

(a) Derive the condition for the extremum of $J = \int_{x_1}^{x_2} f(y, \dot{y}, x)dx$. (06)

(b) Using calculus of variations discuss brachistochrone problem. (06)

OR

(b) Lagrangian of a system is given by $L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{r^2}$. Compute all generalized momenta and energy function. Which of them are conserved? Why?

Q-5

(a) State Hamilton's modified principle; and derive Hamilton's equations of motion from it. (06)

(b) Giving an example describe Routhian procedure. (06)

OR

(b) Find Hamiltonian corresponding to the Lagrangian,
$L = a\,\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\,\dot{y} + f\,y^2\dot{x}\,\dot{z} + g\,\dot{y} - k\sqrt{x^2 + y^2}$,
where a, b, c, f, g and $k$are constants; x, y and z are generalized coordinates.

Q-6

(a) Define fundamental Lagrange brackets. Show that they are invariant under a canonical transformation. (06)

(b) Define infinitesimal canonical transformation. Show that symplectic condition is satisfied in this case. (06)

OR

(b) Show that the transformation,
$Q = \log(1 + \sqrt{q}\cos p)\,, P = 2\sqrt{q}\,(1 + \sqrt{q}\cos p)\sin p$,
is canonical.

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