(31A & A-26) **Seat No.:** ________ No of printed pages : 3

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - VI) EXAMINATION - 2017
Tuesday , 28^{th} March , 2017
MATHEMATICS : US06CMTH02
(COMPLEX ANALYSIS)

Time : 10:00 a.m. to 1:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) $f(z) = (x^2 - y^2 - 2y) + i(2x - 2xy)$ can be expressed as $f(z) =$..........................

(a) $\bar{z}^2 + 2z$ (b) $\bar{z}^2 + 2iz$ (c) $\bar{z}^2 - 2iz$ (d) $\bar{z}^2 + iz$

(2) $\lim_{z\to 0} \frac{z}{\bar{z}} =$ ....

(a) -1 (b) 0 (c) 1 (d) does not exist

(3) $f(z) = \frac{2z}{z(z^2+1)}$ is analytic in ....

(a) $\mathbb{C} - \{\pm i\}$ (b) $\{0\ ,\ \pm i\}$ (c) $\mathbb{C} - \{0\}$ (d) $\mathbb{C} - \{0\ ,\ \pm i\}$

(4) If C-R equations are satisfied at z_0 then f(z) is .... at z_0 .

(a) need not be differentiable (b) not differentiable (c) differentiable (d) none of these .

(5) $exp(2 \pm 3\pi i) =$ ....

(a) e^{-2} (b) e^2 (c) $-e^2$ (d) $-e$

(6) $e^{z_1} = e^{-z_2}$ then $z_1 =$ ....

(a) $-z_2 + 2n\pi i$ (b) $z_2 + 2n\pi$ (c) $-z_2$ (d) z_2

(7) $cosiy =$ ....

(a) $icoshy$ (b) $coshy$ (c) $-coshy$ (d) $icosy$

(8) The image of line $x = c_1$, $c_1 \neq 0$ under the transformation $w = 1/z$ is ....

(a) circle (b) square (c) rectangle (d) hyperbola

(9) Fixed point of $w = \frac{6z-9}{z}$ is ....

(a) 0 (b) i (c) 2 (d) 3

(10) Image of $x < 0$ under the transformation $w = (1+i)z$ is ....

(a) $u < v$ (b) $v < u$ (c) $u < -v$ (d) $u > -v$

Que.2 Answer the following (Any Ten) 20

(1) Prove that limit of function is unique,if it exist.

(2) By using definition prove that $\lim_{z\to z_0} (z^2 + c) = {z_0}^2 + c$, where c is complex constant.

(3) If f and g are differentiable then prove that fg is differentiable .

(4) Prove that $f(z) = e^{ix+y}$ is nowhere analytic .

 (P.T.O.)

(5) Check whether $f(z) = \cosh x \cos y + i \sinh x \sin y$ is entire or not .Verify it .

(6) Prove that $e^{-y} \sin x$ and $-e^{-y} \cos x$ are harmonic at each point of domain of xy-plane.

(7) Prove that $|exp(-2z)| < 1$ iff $Rez > 0$.

(8) Prove that $\dfrac{d}{dz}(\coth z) = -\operatorname{cosech}^2 z$.

(9) Find all values of $\sinh^{-1} z = log(z + \sqrt{z^2+1})$.

(10) Prove that the general linear transformation $w = Az + B$, $A \neq 0$, A and B are complex constant, gives expansion or contraction and a rotation followed by a translation.

(11) Find image of $a \leq x \leq b$; $0 \leq y \leq \pi$ under the transformation $w = e^z$.

(12) For the transformation $w = \sin z$,prove that a line $x = c_1$, $(0 < c_1 < \pi/2)$ is mapped onto the right hand branch of the hyperbola $\dfrac{u^2}{\sin^2 c_1} - \dfrac{v^2}{\cos^2 c_1} = 1$.

Que.3 (a) Give an example of function such that its real and imaginary component have continuous partial derivative of all order at a point but the function is not differentiable at that point.Verify it. 6

(b) If $f(z) = \dfrac{x^3 y(-y+ix)}{z(x^6+y^2)}$, $z \neq 0$, $f(0) = 0$ 4

(i) Is $\lim\limits_{z \to 0} f(z)$ exists ? (ii) Is $f(z)$ differentiable at 0 ? . Verify it .

OR

Que.3 (a) State and prove chain rule for differentiating composite functions. 6

(b) By using definition of limit prove that $\lim\limits_{z \to (1-i)} (x + i(2x+y)) = 1 + i$. 4

Que.4 (a) State and prove sufficient conditions for differentiability of $f(z)$. 5

(b) Let $f(z) = u(x,y) + iv(x,y)$ and $f'(z)$ exist at $z_0 = x_0 + iy_0$.Prove that the first order partial derivatives of u and v must exist at (x_0, y_0) and they satisfies the Cauchy-Reimann equations $u_x = v_y$; $u_y = -v_x$ at (x_0, y_0). 5

OR

Que.4 (a) Find a harmonic conjugate $v(x,y)$ for harmonic function $u(x,y) = y^3 - 3x^2 y$. 4

(b) For $f(z) = iz + 2$,prove that $f'(z)$ and $f"(z)$ exist everywhere. 3

(c) Give an example of function which is analytic at every non-zero points.Verify it. 3

Que.5 (a) Prove that $\overline{exp(iz)} = exp(i\bar{z})$ iff $z = n\pi$, $n \in \mathbb{Z}$. 4

(b) Solve the equation $sinhz = i$. 3

(c) Evaluate $log(-\sqrt{3} - i)$ and $Log(-1 - \sqrt{3}i)$. 3

OR

Que.5 (a) Describe $\cos^{-1} z$ in terms of logarithm .Hence find all values of $\cos^{-1}(\sqrt{2})$. 5

(b) Prove that $|\cos z|^2 = \cos^2 x + \sinh^2 y$. 3

(c) Prove that $|\sinh x| \leq |\cosh z| \leq \cosh x$. 2

Que.6 (a) Prove that all linear fractional transformation that maps the upper half plane $Im\ z > 0$ on to the open disk $|w| < 1$ and the boundary $Im\ z = 0$ on to the boundary of $|w| = 1$ is given by $w = e^{i\alpha}\left[\dfrac{z - z_0}{z - \overline{z_0}}\right]$, $Im\ z_0 > 0$. Also prove the converse. 6

(b) Find linear fractional transformation that maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ *on to* $w_1 = 1$, $w_2 = i$, $w_3 = -1$ respectively . 4

OR

Que.6 (a) Discuss the image of $w = (i + 1)z + 2$. Hence sketch the rectangle $1 \leq x \leq 2$, $1 \leq y \leq 4$ and its image. 5

(b) Find the image of semi infinite strip $x > 0$, $0 < y < 1$ under the transformation $w = i/z$. Also sketch the strip and its image. 5

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