(A-10) Seat No: ________ No. of Printed Pages : 3 SC

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Subject : Mathematics US05CMTH01 Max. Marks : 70
Date: 09/05/2016 Real Analysis-I
Timing: 10.30 am - 01.30 pm

Instructions : (1) This question paper contains SIX QUESTIONS
(2) The figures to the right side indicate full marks of the corresponding question/s
(3) The symbols used in the paper have their usual meaning, unless specified

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Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] If the greatest member of a set exists then the set is

[A] bounded [B] unbounded [C] bounded below [D] bounded above

[2] The greatest and the smallest members of the set $\{0\}$

[A] are equal [B] are not equal [C] do not exist [D] none

[3] The smallest member of a set S, if exists, is

[A] the supremum of S [B] the infimum of S [C] not unique [D] none

[4] If $S = (1,5) - \{3\}$, then 3 is

[A] a limit point of S
[B] an interior point of S
[C] interior point as well as limit point of S
[D] none

[5] If S_1 and S_2 are closed sets then $S_1 \cup S_2$ is

[A] closed [B] open [C] Open as well as closed [D] none

[6] The interior of the set of integers is

[A] N [B] Q [C] R [D] ϕ

[7] If $\lim_{x\to a} f(x)$ exists but $f(a)$ does not exist then f posseses a discontinuity of

[A] removable type [B] first type
[C] second type [D] first type from left

[8] If a function $f(x)$ has a discontinuity of first type at $x = 2$ then $\lim_{x\to 2-} f(x)$ and $\lim_{x\to 2+} f(x)$ both

[A] do not exist [B] exist and they are equal
[C] exist but they are not equal [D] cannot exist togather

[9] The condition that f is monotonic increasing at c is

[A] $f'(c) = 0$ [B] $f'(c) \neq 0$ [C] $f'(c) \geqslant 0$ [D] $f'(c) \leqslant 0$

[10] If f is continuous on an interval I then

[A] f is uniformly continuous on I
[B] f is not necessarily uniformly continuous on I
[C] f may have some points of discontinuities in I
[D] none

Q: 2. Answer any TEN of the following. **20**

[1] Find the g.l.b and greatest member of $\left\{\frac{1}{n^3} / n \in N\right\}$ if they exist.

[2] Define Complete Ordered Field.

[3] Find the supremum and the infimum of the set $\{1\}$, if they exist.

[4] Find the set of all the interior points of $\{1, 2, 3, e, \pi\}$

[5] Give an example of a set which is neither open nor closed.

[6] Find the largest open subset of $(1, 2) \cup (4, 8)$.

[7] Examine the function

$$f(x) = \begin{cases} 2x + 1 & \text{when } x \neq 1 \\ 3, & \text{when } x = 1 \end{cases}$$

for continuity at $x = 1$

[8] If [x] denotes the largest integer less than or equal x, then discuss the continuity at $x = 3$ for the function $f(x) = x - [x]$ defined for all $x \geqslant 0$.

[9] Evaluate $\lim\limits_{x \to 0-} \dfrac{e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}}$ if it exists.

[10] Examine whether the function $f(x) = \begin{cases} 2 & \text{if } 0 \leqslant x < 1 \\ 2x & \text{if } x \geqslant 1 \end{cases}$ is differentiable at $x = 1$ or not

[11] Prove that the function $f(x) = |x|$ is not derivable at origin

[12] Prove that the function x^2 is derivable on $(0, 1)$.

Q: 3. State Least Upper Bound property of R and prove that the set of rational numbers is not order complete. **10**

OR

Q: 3 [A] For all real numbers x and y, prove the follwing :

(i) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ where $y \neq 0$

(ii) $||x| - |y|| \leqslant |x - y|$ **6**

[B] If a is a positive real number and b is any real number then prove that there exists a positive integer n such that $na > b$. **4**

Q: 4 [A] Show that a set is closed iff its complement is open **6**

[B] Show that the union of two closed sets is closed **4**

OR

Q: 4 [A] Define Interior of a set and show that the interior of a set contains every open subset of a set. **6**

[B] Show that the intersection of two neighborhoods is also a neighborhood. **4**

Q: 5. If a function f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs, then prove that there exists at least one point $\alpha \in (a, b)$ such that $f(\alpha) = 0$. **10**

OR

Q: 5 [A] Prove that limit of a function is unique, if exists. **6**

[B] If a function f is continuous at an interior point c of $[a, b]$ and $f(c) \neq 0$, then prove that there exists $\delta > 0$ such that $f(x)$ has the same sign as $f(c)$ for every $x \in (c - \delta, c + \delta)$. **4**

Q: 6 [A] Show that a function which is derivable at a point is necessarily continuous at that point. What about converse? **6**

[B] Prove that $\frac{x}{1+x} < \log(1 + x) < x$ for all $x > 0$. **4**

OR

Q: 6 [A] If $f'(c) < 0$, then prove that f is monotonic decreasing function at point $x = c$. **6**

[B] If f is derivable at c and $f(c) \neq 0$ then prove that $\frac{1}{f}$ is also derivable thereat and $\left(\frac{1}{f}\right)' = -\frac{f'(c)}{\{f(c)\}^2}$. **4**

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