SEAT No.________

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B.Sc. EXAMINATION (SEMESTER: IV)

2017

[63/A-32]

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Tuesday

Subject: Probability Distributions

Subject code: US04CSTA02

Time: 02.00 p.m. to 05.00 p.m. **Marks: 70**

1 **Multiple Choice Questions** [10]

(1) For a binomial distribution with n = 7 and p = 0.50, we get ------

(a) P(X = 5) < P(X = 2) (b) P(X = 5) > P(X = 2)

(c) P(X = 5) = P(X = 2) (d) P(X = 5) ≠ P(X = 2)

(2) The recurrence relation for the probability of geometric distribution with parameter p is f(x + 1) = --------.

(a) q.f(x) (b) p . f(x) (c) f(x) + p (d) none

(3) If $M_X(t) = e^{25t(1+2t)}$ is the m.g.f. of a continuous random variable X then mean and standard deviation of X is --------- and --------..

(a) 25 and 10 (b) 25 and 100 (c) 25 and 50 (d) none

(4) If $M_X(t) = (1-5t)^{-1}$, t ≠ 0, is the m.g.f. of a random variable X then X follows -------- distribution

(a) normal (b) exponential (c) continuous uniform (d) none

(5) If $f(x) = kx^2$, 0 < x < 1. and zero , otherwise ; is the p.d.f. of X then k = ---------.

(a) 2 (b) 3 (c) 4 (d) 5

(6) If X ~ b(n,p) distribution then as $n \to \infty$ $\frac{X-np}{\sqrt{npq}}$ follows approximately-------------- distribution.

(a) binomial (b) Poisson (c) standard normal (d) none

(7) If X_1 and X_2 are two independent exponential variate with mean θ each, then $Y = X_1 + X_2 \sim$ --------- distribution.

(a) G(2,θ) (b) G(2,2θ) (c) G(1,θ) (d) none

(8) If X_1 and X_2 are two independent N(2,5) and N(3,4) distributions respectively then the distribution of $Y = X_1 + X_2$ follows ---------- distribution.

(a) N(5,8) (b) N(5,10) (c) N(5,9) (d) none

(9) If $X_i \sim NID(\mu, \sigma^2)$ distribution for i = 1,2,...... 25 then $\bar{X} \sim$ ---------- distribution

(a) $N(\mu, 25\sigma^2)$ (b) $N(25\mu, 25\sigma^2)$ (c) $N(\mu, \frac{\sigma^2}{25})$ (d) none

(10) If $S^2 = \frac{1}{n}\sum_1^n (X_i - \bar{X})^2$ is the sample variance of a random sample from $N(\mu, \sigma^2)$ then $\frac{(n-1)S^2}{\sigma^2} \sim$ --------- distribution.

(a) $\chi^2_{(n)}$ (b) $\chi^2_{(n-1)}$ (c) $\chi^2_{(1)}$ (d) none

2 **Short Questions (Attempt any TEN)** [20]

(1) Define Hyper geometric distribution. State its mean and variance.

(2) Define Poisson distribution. State its mean, variance and its p.g.f.

Show that the recurrence relation is given by $f(x+1) = \frac{\lambda}{x+1} f(x)$.

(3) If $P(t) = (\frac{2}{3} + \frac{1}{3}t)^9$ is the p.g.f. of a random variable X then identify the distribution of X and state its mean, variance p.m.f. and m.g.f.

(4) If $X \sim N(50, 36)$ distribution then find (i) $P(32 \leq X \leq 62)$ and $P(|X - 50| < 6)$.

(5) If $f(x) = kx(1-x)$, $0 < x < 1$ and zero otherwise, is the p.d.f of X then find k and identify the distribution and $P(X < 0.50)$.

(6) If $f(x) = k\,e^{-3x}$, $0 < x < \infty$ and zero otherwise then find k and c.d.f. of X. Name the distribution of X.

(7) If X follows continuous uniform distribution with mean 3 and variance 3, find $P(X > 0)$ and $P(X < 1)$.

(8) If X and Y are independent random variables and X and Y follows Poisson distribution respectively P(5) and P(3). Find the distribution of $Z = X + Y$. Write the p.m.f. of X and find $P(X < 4)$ and $P(X > 5)$.

(9) If $X \sim b(64, 1/2)$ then find $P(X = 36)$ and $P(28 < X < 44)$. State the result you used.

(10) Define F – distribution. Write the p.d.f. of $F_{(2,2)}$ distribution.

(11) Define Chi square distribution. State it's mean, variance and m.g.f.

(12) If $X \sim N(0, 1)$ distribution then state the distribution of X^2 and hence find $P(X^2 > 3.841)$ and $P(X^2 < 6.635)$.

3 (a) Define Negative binomial distribution. State its mean and variance. If a random variable X follows a negative binomial distribution with mean = 12 and variance = 36 then find the p.m.f. of X and also the $P(X = 0)$ and $P(X \geq 1)$. [5]

(b) Define discrete uniform distribution. Find its mean and variance. [5]

OR

3 (a) Define a Poisson Distribution. State its mean, variance and m.g.f. [5]
A car hire firm has two cars, which he hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which (i) neither car is used, and (ii) the proportion of days on which some demand is refused.

(b) If $P(t) = (\frac{2}{5} + \frac{3}{5}e^t)^{10}$ is the m.g.f. of a random variable X then identify the distribution of X and find it's mean, variance . Also find $P(X = 0)$ and $P(X \geq 2)$. [5]

4 (a) Define normal distribution. Obtain its m.g.f. and hence or otherwise find its mean and variance. [5]

(b) If $f(x) = 30x^2(1-x)^2$, $0 < x < 1$; [5]
= 0 , elsewhere
then identify the distribution of X. Find it's mean, harmonic mean and variance

OR

4 (a) If $f(x) = \frac{1}{2a}$, $-a < x < a$ [5]
= o , otherwise . Find the m.g.f. of X. Also prove that all the odd order moments are zero. Find an expression for even order moments

(b) `Define an exponential distribution and, then for every constant $a > 0$, prove that $P[X \leq x + a / X \geq a] = P[X \leq x]$ for all x. [5]

5 (a) If $X_1, X_2, \ldots X_n$ are independent random variables with m.g.f.s. $Mx_1(t)$, $Mx_2(t)$,....... $Mx_n(t)$ then prove that the m.g.f. of $Y = \sum_1^n X_i$ is given by $My(t) = \prod_1^n Mx_i(t)$. [5]

(b) $X \sim b(11, 0.35)$ and $Y \sim b(5, 0.35)$. If X and Y are independent, Find the distribution [5] of Z = X + Y Find (i) P(X+Y = 6), (ii) P(3 < X+Y ≤ 8).

OR

5 **(a)** If X_1, X_2 are independent random variables with N(0,25) and N(0,144) then find the distribution of Y = X_1 - X_2. Also state its p.d.f. mean and variance of Y. Also find P(Y< 21) and P (|Y|> 21). [5]

(b) X follows Poisson distribution with parameter 100 then find P(X =120) and P(120 ≤ X ≤ 130). State the result you use. [5]

6 **(a)** Define Student's t distribution. X is distributed like Students t with 15 d.f. Find (i) P(0.536 < x < 1.341), (ii) P(|x | > 1.07), (iii) P(X^2 > 4.54), (iv) c such P(|x | < c) = 0.60 [5]

(b) If X_1, X_2 ,……. X_n denote a random sample of size n from a population having mean μ and variance σ^2. If $\bar{X} = \frac{1}{n}\sum_1^n X_i$ and $S^2 = \frac{1}{n}\sum_1^n (X_i - \bar{X})^2$ and $S'^2 = \frac{1}{n-1}\sum_1^n (X_i - \bar{X})^2$. Obtain E($S^2$) and E($S'^2$) [5]

OR

6 **(a)** Define F distribution on (r_1 , r_2) degrees of freedom. If X ~ $F_{(2,r)}$ distribution, r ≥ 2 then prove that P(X ≥ k) = $(1 + \frac{2k}{r})^{-\frac{r}{2}}$ [5]

(b) A random sample of size n = 36 is to be taken from N(15,144) distribution. Find P (13 ≤ $\bar{X}$ ≤ 17) and P ($\bar{X}$ ≤ 19) . Also state E($\bar{X}$), E($\bar{X} + 5$), and V($\bar{X}$), V($\bar{X} - 2$), State the result you use clearly. [5]

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