SARDAR PATEL UNIVERSITY
B. Sc. (Sem. IV) Examination

Saturday, $13^{\text {th }}$ April 2013
11.00 am - 2.00 pm

## US04CMTH02 - Mathematics (Differential Equations)

Total Marks: 70
Note: Figures to the right indicate marks to the questions.
Q. 1 Choose the most appropriate option for the following questions and write it down in the answer book.
(1)

Integral curve of $\frac{d x}{x}=\frac{d y}{y}=\frac{d z}{z}$ is $\qquad$ .
(a) $x=c_{1} y ; y=c_{2} z$
(b) $x-y=c_{1} ; x-z=c_{2}$
(c) $x y=c_{1} ; y z=c_{2}$
(d) $y z+z x+x y=0$
(2) Orthogonal trajectories of given curves $\qquad$ .
(a) Intersect them at acute angle
(b) Intersect them at obtuse angle
(c) Intersect them at right angle
(d) are parallel to those curves
(3) Integral curve of $2 d x=5 d y=7 d z$ is $\qquad$ .
(a) $2 x+5 y+7 z=c$
(b) $x^{2}=y^{5}, x^{2}=z^{7}$
(c) $2 x-5 y=c_{1}, 2 x-7 z=c_{2}$
(d) $2 x+5 y=c_{1}, 5 y+7 z=c_{2}$
(4) The general form of linear partial differential equation is $\qquad$ .
(a) $P p-Q q=R$
(b) $p+q=1$
(c) $p q=1$
(d) $P p+Q q=R$
(5) $a x-b y+z=8$ is a solution of $\qquad$ -.
(a) $p q-q y+z=8$
(b) $q x-p y-z=8$
(c) $p q+q y-z=-8$
(d) $p x+q y-z=-8$
(6) Eliminating the arbitrary constants from $z=(x+a)(y+b)$ we get $\qquad$ .
(a) $p+q=z$
(b) $p-q=z$
(c) $p q=z$
(d) $\frac{p}{q}=z$
(7) The surface orthogonal to one parameter family of surface $f(x, y, z)=c$ are the surface generated by the integral curve of the equations $\qquad$ .
(a) $\frac{d x}{\frac{\partial f}{\partial x}}=\frac{d y}{\frac{\partial f}{\partial y}}=\frac{d z}{\frac{\partial f}{\partial z}}$
(b) $\frac{d x}{\frac{\partial f}{\partial z}}=\frac{d y}{\partial f}=\frac{d z}{\partial x} \frac{d f}{\partial y}$
(c) $\frac{d x}{\frac{\partial f}{\partial y}}=\frac{d y}{\frac{\partial f}{\partial z}}=\frac{d z}{\frac{\partial f}{\partial x}}$
(d) $\left(\frac{\partial f}{\partial x}\right) d x=\left(\frac{\partial f}{\partial y}\right) d y=\left(\frac{\partial f}{\partial z}\right) d z$
(8) Let $F(u, v)=0$, where $u=y-x=c_{1}$ and $v=z-x=c_{2}$ be the general solution of $p+q=1$ then the solution passing through the curve $x=0, y^{2}=z$ is
$\qquad$ .
(a) $(y-x)^{2}=z$
(b) $(y-x)^{2}=z-x$
(c) $y-x=(z-x)^{2}$
(d) None
(9) Two differential equations $f(x, y, z, p, q)=0$ and $g(x, y, z, p, q)=0$ are compatible is $\qquad$ .
(a) $[f, g]=0$
(b) $[f, g]=1$
(c) $[f, g]=p$
(d) $[f, g]=q$
(10) For linear partial equation with constant coefficient $F\left(D, D^{\prime}\right) z=f(x, y)$ the operator $\mathrm{D}^{\prime}=$ $\qquad$ .
(a) $\frac{\partial}{\partial x}$
(b) $\frac{\partial}{\partial p}$
(c) $\frac{\partial}{\partial q}$
(d) $\frac{\partial}{\partial y}$
Q. 2 Answer the following questions in short. (Attempt Any Ten)
(1) Find the integral curves of the equation $x d x=y d y=z d z$
(2) Solve: $\frac{d x}{2 x z}=\frac{d y}{2 y z}=\frac{d z}{z}$
(3) Solve: $\frac{d x}{1+x}=\frac{d y}{1+y}=\frac{d z}{z}$
(4) Verify whether the differential equation $y d x+x d y-z d z=0$ is integrable or not.
(5) Eliminate arbitrary function ' $f$ ' from the equation $z=x y+f\left(x^{2}+y^{2}\right)$
(6) Obtain partial differential equation of $a x-b y+4 z=12$
(7) Obtain partial differential equations of the form $\frac{d x}{P}=\frac{d y}{Q}=\frac{d z}{R}$ whose integral curves generate surfaces orthogonal to the surfaces $5 x^{2}+6 y^{2}+7 z^{2}=c$.
(8) Find integral surface of $x^{2}+y=c_{1}, x z+y=c_{2}$ passes through the line $x=0, y=1$.
(9) Find the differential equation of the surface which is orthogonal to $x^{2}+y^{2}+z^{2}=c z$
(10) Find the general solution of the equation $2 p+q=3$
(11) Find the complete integral of $p q=1$
(12) If $z=t x+y f(t)+g(t)$ then prove that $r t-s^{2}=0$
Q. 3
(a) Solve: $\frac{d x}{x+z}=\frac{d y}{y}=\frac{d z}{z+y^{2}}$
(b) Find the orthogonal trajectories on the cone $x^{2}+y^{2}=z^{2} \tan ^{2} \alpha$ of its intersection with the family of planes parallel to $z=0$.

## OR

Q. 3
(a) Solve: $\frac{d x}{y(x+y)+a z}=\frac{d y}{x(x+y)-a z}=\frac{d z}{z(x+y)}$
(b) Solve: $\frac{d x}{x\left(y^{2}-z^{2}\right)}=\frac{d y}{y\left(z^{2}-x^{2}\right)}=\frac{d z}{z\left(x^{2}-y^{2}\right)}$
Q. 4
(a) Integrate: $2 x z d x+z d y-d z=0$
(b) In usual notations prove that $p \frac{\partial(u, v)}{\partial(y, z)}+q \frac{\partial(u, v)}{\partial(z, x)}=\frac{\partial(u, v)}{\partial(x, y)}$

## OR

Q. 4
(a) Find the general solution of given linear partial differential equation

$$
\begin{equation*}
x^{2} \frac{\partial z}{\partial x}+y^{2} \frac{\partial z}{\partial y}=(x+y) z \tag{05}
\end{equation*}
$$

(b) If $X$ is a vector such that $X \cdot \operatorname{curl} X=0$ and $\mu$ is an arbitrary function of $\mathrm{x}, \mathrm{y}, \mathrm{z}$ then prove that $(\mu X) \cdot \operatorname{curl}(\mu X)=0$
Q. 5
(a) Find the integral surface of the equation
$(x-y) y^{2} p+(y-x) x^{2} q=\left(x^{2}+y^{2}\right) z$ which passes through $x z=a^{3}, y=0$.
(b) Find the surface which intersects the surfaces of the system $z(x+y)=c(3 z+1)$ orthogonally and which passes through the circle $x^{2}+y^{2}=1, z=1$.

## OR

Q. 5
(a) Show that $(x-a)^{2}+(y-b)^{2}+z^{2}=1$ is the complete integral of nonlinear p.d.e. $z^{2}\left(1+p^{2}+q^{2}\right)=1$. Determine a general solution by finding the envelope of its particular solution.
(b) Find the integral surface of the linear
p.d.e. $x\left(y^{2}+z\right) p-y\left(x^{2}+z\right) q=\left(x^{2}-y^{2}\right) z$ which contains the straight line $x+y=0, z=1$.
Q. 6 Prove that the equation $f(x, y, p, q)=0$ and $g(x, y, p, q)=0$ are
compatible if $\frac{\partial(f, g)}{\partial(x, p)}+\frac{\partial(f, g)}{\partial(y, q)}=0$ and verify that the equations $p=P(x, y)$ and $q=Q(x, y)$ are compatible if $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$. Also find complete integral of $p+q=p q$.

## OR

Q. 6 If $\mu_{1}, \mu_{2} \ldots \mu_{n}$ are solutions of homogeneous linear partial differential
equation $F\left(D, D^{\prime}\right) z=0$ then prove that $\sum_{r=1}^{n} c_{r} \mu_{r}$ is also solution of $F\left(D, D^{\prime}\right) z=0$ where $c_{r}$ are arbitrary constants. Hence solve $\frac{\partial^{4} z}{\partial x^{4}}+\frac{\partial^{4} z}{\partial y^{4}}+2 \frac{\partial^{4} z}{\partial x^{2} \partial y^{2}}$.

