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SARDAR PATEL UNIVERSITY
B.Sc. (SEM-IV) Examination(Regular & NC)
Wednesday, 13th April, 2016
US04EMTH05: (Calculus and Algebra-2)

Time: 10:30 a.m. to 12:30 p.m. **Maximum Marks : 70**

Note: Figures to the right indicate marks to the questions.

Q.1 Answer the following by selecting the correct choice from the given options. **[10]**

(1) If AC-$B^2 = 0$, Then _____
(a)local minimum and local maximum of f do not exist (b)we can not conclude anything
(c) f(a,b) is a local maximum of f (d) f(a,b) is a local minimum of f

(2) If f(x,y) ≤ f(a,b) for all (x,y)∈ E ⊂ R^2, then f is said to be _______
(a) local extreme at (a,b) (b)global minimum at (a,b)
(c) global maximum at (a,b) (d)constant value at (a,b)

(3) If $f(x,y) = x^3 + y^3$, grad.f = _______
(a)$3x^2\bar{\imath} + 2y\bar{\jmath}$ (b)$3x^2\bar{\imath} - 2y\bar{\jmath}$ (c)$3x\bar{\imath} - 2y\bar{\jmath}$ (d)$3x^2\bar{\imath} + y^2\bar{\jmath}$

(4) $\nabla(\nabla f)$=_______
(a) 0 (b) $(\nabla f)^2$ (c) $2\nabla f$ (d) $\nabla^2 f$

(5) Divergence of $\bar{v}$ is denoted by _______
(a) $\nabla \times \vec{v}$ (b)$\nabla\vec{v}$ (c)$\nabla \cdot \vec{v}$ (d)$\nabla^2 \cdot \vec{v}$

(6) If $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$, then $\frac{\partial r}{\partial y}$=______
(a) $\frac{y}{r}$ (b) 1 (c) $\frac{r}{y}$ (d) y

(7) If $\nabla \cdot V = 0$ Then V is called ______
(a) Laplacian (b) Harmonic (c) Irrotational (d) Solenoidal

(8) $\nabla \times (\nabla f)$=________
(a) -1 (b) $\bar{0}$ (c) 1 (d) 0

(9) For an element 'a' in Boolean Algebra $a \cdot 0 =$______
(a) a (b) 1 (c) 0 (d) a'

(10) If $a \in B$. Then a+a'=______
(a) a (b) a' (c) 0 (d) 1

Q.2 Answer ANY TEN of the following: **[20]**

(1) Define stationary point.
(2) Find A, B ,C for the function $x^4 - 2x^2 - 2y^2 + 4xy + y^4$.
(3) Find stationary point of $y^2 + x^2y + x^4$
(4) Define : Gradient of scalar field.
(5) Find Gradient of a function $f(x,y) = log r$, where $\bar{r} = x\bar{\imath} + y\bar{\jmath}$
(6) Prove that $\nabla(f+g) = \nabla f + \nabla g$.
(7) Define: The divergence of a vector field.
(8) Prove that $\nabla \times (f\bar{v}) = f(\nabla \times \bar{v}) + \nabla f \times \bar{v}$.
(9) Define : The curl of a vector field.
(10) Write distribution law for Boolean Algebra.
(11) Write any two laws of compliment in Boolean Algebra.
(12) Draw the network represented by following Boolean algebra.
$$x(xy + x' + xy')$$

(P.T.O.)

Q.3 Investigate the maxima and minima of the function $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$. (10)

OR

Q.3 A rectangular box open at the top is to have a volume of $32\ m^2$. Find the dimension of box so that the total surface area is minimum. (10)

Q.4

(a) Find gradient of the function $f(x,y) = (x^2 + y^2 + z^2)^2$ at (1,2,3). (5)

(b) Show that the function $f(x,y) = \frac{x}{(x^2+y^2)}$ is a harmonic function. (5)

OR

Q.4

(a) Find directional derivative of $f(x,y,z) = 4xz^3 - 3x^2y^2z\ at\ (2,-1,2)$ in the direction of $\bar{a} = 2\bar{i} - 3\bar{j} + 6\bar{k}$. (5)

(b) Find unit normal vector to the surface $z^2 = (x^2 + y^2)\ at\ (3,4,5)$. (5)

Q.5

(a) Show that $\nabla(\bar{r}^n\bar{r}) = (n+3)\bar{r}^n$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, $r = |\bar{r}|$. (5)

(b) Show that $\nabla(\nabla \times \bar{v}) = 0$. (5)

OR

Q.5

(a) Verify $\nabla(f\bar{v}) = f(\nabla\bar{v}) + \bar{v}\nabla f$ for $f = e^{xyz}$ and $\bar{v} = ax\bar{i} + by\bar{j} + cz\bar{k}$. (5)

(b) Show that $\nabla(\frac{r}{r^3}) = 0$. (5)

Q.6

(a) In every Boolean algebra (B) each of the binary operations (+) and (·) are associative. (5)

(b) Simplify following function and draw the network represented by them. (5)

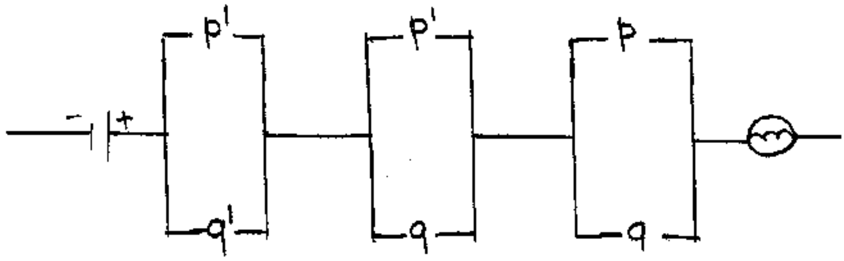
(1) 1+x (2) $x \cdot 0$ (3) $x + xy'$

OR

Q.6

(a) State and prove De-Morgan's law for Boolean algebra (B). (5)

(b) Find Boolean function for the following circuit. Simplify it and draw simplified circuit. (5)



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