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[59/A-22]

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - IV) EXAMINATION - 2017
Saturday , 15th April , 2017
MATHEMATICS : US04EMTH01
(Boolean Algebra and Laplace Transforms)

Time : 02:00 p.m. to 04:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) $a.(a+b)$ = ...

(a) b (b) a (c) a+b (d) a.b

(2) $a+(a.b)$ = ...

(a) a (b) b (c) $a.b$ (d) $a+b$

(3) Initial approximation of $x^3 - x - 2 = 0$ can be chosen from .....

(a) [0,1] (b) [-1,0] (c) [1,2] (d) [-2,-1]

(4) Aitken's Δ^2 process is used for finding approximation .....

(a)Derivative of a function (b) Integral of a function (c) Root of equation (d) None

(5) In Bisection method, $x^3 - 9x + 1 = 0$; a=2 and b=3 then x_0=.....

(a) 2 (b) 3 (c) 2.5 (d) 1.5

(6) L[$cosat$]=.....

(a) $\frac{s}{s^2+a^2}$ (b) $\frac{a}{s^2+a^2}$ (c) $\frac{a}{s^2-a^2}$ (d) $\frac{s}{s^2-a^2}$

(7) L[$sinhat$]=.....

(a) $\frac{a}{s^2-a^2}$ (b) $\frac{a}{s^2+a^2}$ (c) $\frac{s}{s^2+a^2}$ (d) $\frac{s}{s^2-a^2}$

(8) $L^{-1}[\frac{s}{s^2+a^2}]$=.....

(a) $\cos at$ (b) $\cosh at$ (c) $\frac{1}{a}\cosh at$ (d) $a\cosh at$

(9) If $L^{-1}\{f(s)\} = f(t)$, then $L^{-1}\{f(s-a)\}$=.....

(a) $e^{at}f'(t)$ (b) $e^{at}f(t)$ (c) $f(t)$ (d) None

(10) $L^{-1}[\frac{1}{s^2}]$=.....

(a) 1 (b) t^2 (c) t (d) t^3

Que.2 Answer the following (Any ten) 20

(1) Prove that the element a' associated with element a in a Boolean algebra is unique .

(2) Define Boolean Algebra and state its Properties.

(3) State Principal of duality.

(4) Solve the equation $f(x) = e^x - 3x = 0$, by using Bisection Method.

(5) Define Algebraic and Transcendental Equation with example .

(6) Using Newton Raphson Method find the real root of the equation $\sin x = \frac{x}{2}$.

(7) Prove that $L(e^{at}) = \frac{1}{s-a}$, $s > a$.

(8) Find laplace transform of $t\cos 3t$.

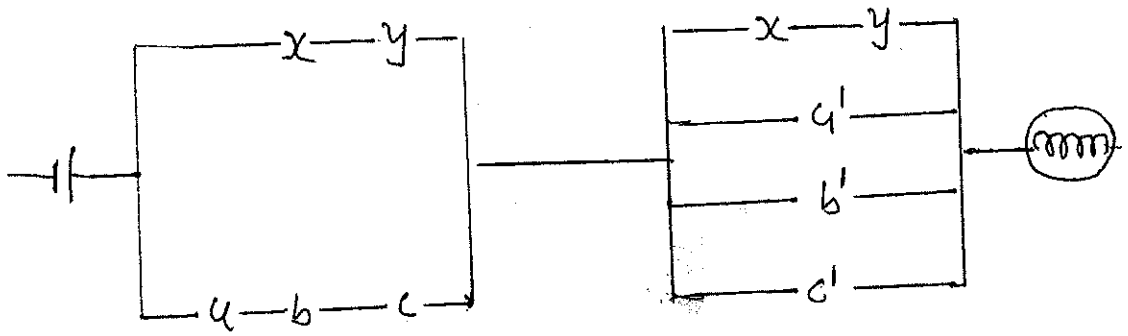
(9) Find laplace transform of $\cos^2 2t$.

(10) Evaluate $L^{-1}\left[\frac{1}{s}\right] = 1$.

(11) Evaluate $L^{-1}[\frac{1}{s^2+a^2}] = \frac{1}{a}\sin at$.

(12) Evaluate $L^{-1}[\frac{s}{s^2-a^2}] = \cosh at$.

Que.3 (a) Find the Boolean function of switching circuit given below and simplify it . Also draw the simplified circuit.

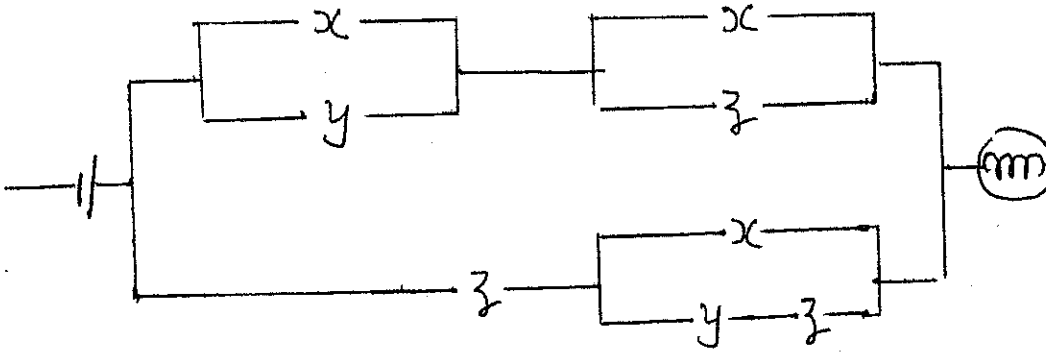


(b) If a and b are elements of boolean algebra B,satisfying the relation $a \leqslant b$ then prove that $a + bc = b(a + c)$, $\forall\, c \in B$.

(c) If $a + x = b + x$ & $a + x' = b + x'$ then prove that $a = b$.

OR

Que.3 (d) Find the Boolean function of switching circuit given below and simplify it . Also draw the simplified circuit.



(e) Prove that in Boolean algebra , every triple of elements a, b, c satisfies the identity $ab + bc + ca = (a + b)(b + c)(c + a)$.

(f) Prove that for every a & $b \in B$, $(a + b)' = a'b'$.

Que.4 (a) Find the real root of the equation $\sin x = 10(x-1)$,correct up to 3 decimal places by using Iteration Method. 5

(b) Find the real root of the equation $f(x) = x^3 - x - 4 = 0$,correct up to 3 decimal places by using False Position Method. 5

OR

Que.4 (c) Using Newton Raphson formulae, establish the iterative formula $x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$ to calculate the square root of N. Using the formulae find the square root of 8 and 5. 5

(d) Find the real root of the equation $2x = \cos x + 3$,correct up to 3 decimal places by using Aitken's Δ^2 Process. 5

Que.5 (a) If $L\{f(t)\} = f(s)$ then prove that $L\{t^n\ f(t)\} = (-1)^n \frac{d^n}{ds^n}[\bar{f}(s)]$, where $n = 0, 1, 2, \ldots$. 4

(b) Find the laplace transform of $\frac{(1-e^t)}{t}$. 3

(c) Evaluate $\int_0^\infty te^{-2t} \sin t\ dt$. 3

OR

Que.5 (d) Prove that $L(t^n) = \frac{n!}{s^{n+1}}$ Where $n = 0, 1, 2, \ldots$ otherwise $\left[\frac{\lfloor n+1}{s^{n+1}}\right]$. 4

(e) Evaluate $L\left\{\int_0^t \frac{e^t \sin t}{t} dt\right\}$. 3

(f) Find Laplace transform of $t^2 \sin at$. 3

Que.6 (a) Prove that $L^{-1}\left[\frac{1}{(s-a)^2+b^2}\right] = \frac{1}{b}\ e^{at} \sin bt$. 4

(b) Find the inverse Laplace transform of $\frac{1}{s(s+a)^3}$. 3

(c) Find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2-a^2)}$. 3

OR

Que.6 (d) Apply Convolution Theorem to evaluate $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$. 4

(e) Prove that $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \frac{1}{2a} t \sin bt$. 3

(f) Find the inverse Laplace transform of $\frac{s+3}{s^2-4s+13}$, by using shifting Theorem . 3