## SARDAR PATEL UNIVERSITY

## BSc. (III SEM.) (CBCS) EXAMINATION

## Monday, $26^{\text {th }}$ November 2012

### 10.30 am - 1.30 pm

## US03CMTH01 : Mathematics (Advanced Calculus)

Total Marks: 70
Note: Figures to the right indicate full marks.
Q. 1 Answer the following questions by selecting the most appropriate [10] option. Write down the option in your answerbook.
(1) If $\bar{r}(t)=a \cos t \bar{i}+b \sin t \bar{j}$; where $\mathrm{a}, \mathrm{b}$ are constant, then $\bar{r}$ represents
(a) Helix
(b) Parabola
(c) Circle
(d) Ellipse
(2) $\qquad$
(a) -3
(b) 3
(c) $1 / 2$
(d) -1
(3) The line integral of a closed curve is $\qquad$ .
(a) 0
(b) 1
(c) -1
(d) Does not exist
(4) Area of the regin $R$ : $r=a$ is $\qquad$ .
(a) $\pi a^{2}$
(b) $\pi^{2} a^{2}$
(c) $\pi^{2} a$
(d) $a \pi$
(5) Area of plane region in polar form is given by $\mathrm{A}=$ $\qquad$ .
(a) $\int_{C} r^{2} d \theta$
(b) $\frac{1}{2} \int_{C} r^{2} d \theta$
(c) $\int_{C} r d \theta$
(d) $\frac{1}{2} \int_{C} r d \theta$
(6) $\oint_{C} \bar{u} d \bar{r}=0$ if and only if
(a) $\nabla \bar{u}=0$
(b) $\nabla \times \bar{u}=0$
(c) $\bar{u}$ is work less
(d) none of these.
(7) Moment of Inertia of surface $S$ about $x$-axis is denoted by $\qquad$ .
(a) Iy
(b) $\mathrm{I}_{y}$
(c) Ix
(d) $I_{x}$
(8) The unit normal vector to the surface $f(x, y, z)=0$ is $\qquad$ .
(a) $\frac{\bar{\nabla} f}{|\bar{\nabla} f|}$
(b) $\frac{|\bar{\nabla} f|}{\bar{\nabla} f}$
(c) $\frac{\bar{\nabla} f}{|f|}$
(d) $\frac{|\bar{\nabla} f|}{f}$
(9) If f is harmonic function, then $\qquad$ .
(a) $\nabla f=0$
(b) $\nabla^{2} f=0$
(c) $\nabla^{2} \vec{f}=0$
(d) $\nabla \vec{f}=0$
(10) The unit normal vector to the surface $f(x, y, z)=0$ denoted by $\qquad$ .
(a) $n$
(b) N
(c) $\vec{n}$
(d) $\vec{N}$
Q. 2 Write down any answer of Any Ten questions from the following in
[20] short.
(1) Evaluate $\int_{0}^{\pi / 2} \int_{0}^{1} x^{2} y^{2} d x d y$.
(2) Discuss the work done by force.
(3) Write down the formula for moment of inertia about $x$-axis, $y$-axis and origin.
Check whether the line integral $\int_{(2,0,0)}^{(1,2,3)}(x d x+y d y+z d z)$ is independent of path or not.
(5) State Green's theorem for plane.
(6) When a line integral is said to be independent of path?
(7) Describe area of surface $\bar{r}(u, v)$
(8) Evaluate $\iint_{S}[(x+z) d y d z+(y+z) d z d x+(x+y) d x d y]$
where $S: x^{2}+y^{2}+z^{2}=1$
(9) Write down the parametric form of a surface.
(10) State first form of Green's theorem.
(11) In usual notations prove that, $\iiint_{R} \nabla^{2} f d v=\iiint_{S} \frac{\partial f}{\partial n} d A$.
(12) State second form of Green's theorem.
Q. 3
(a) Evaluate $\int_{C} 2 x y^{2} d s$ where C is a circle $\mathrm{x}^{2}+\mathrm{y}^{2}=1$ in xy -plane from a point $(1,0)$ to $(0,1)$.
(b) Find the volume of the region bounded by the cylinder $x^{2}+y^{2}=4$ \& $y+z=4, z=0$.

## OR

Q. 3
(a) Transform $\int_{R} \int\left(x^{2}+y^{2}\right) d x d y$ in $u v$ plane by taking $x+y=u, x-y=v$.

Then evaluate it, where R: parallelogram with vertices $(0,0),(1,1),(2,0)$ (1,-1).
(b) Find volume of the region bounded by the first octant section cut from the region inside the cylinder $x^{2}+y^{2}=1$ and by the planes $y=0, z=0, x=y$.
Q. 4
(a) State and prove Green's theorem in vector form.
(b) Find the area of the region in the first quadrant bounded by $y=x, y+x^{3}$.

## OR

Q. 4
(a) State and prove Green's theorem for plane.
(b) Find the area of the region $\mathrm{R}: \mathrm{r}=\mathrm{a}(1+\cos \theta)$.
Q. 5
(a) State and prove the first fundamental form of a surface in Cartesian form.
(b) Evaluate $\int_{s} \int\left(2 z(x y-x-y) d x d y+x^{2} d y d z+y^{2} d z d x\right)$.

## OR

Q. 5
(a) Evaluate $\int_{s} \int f(x, y, z) d A$ where $f(x, y, z)=\tan ^{-1}\left(\frac{y}{x}\right)$
$S: z=x^{2}+y^{2}, 1 \leq z \leq 4, x \geq 0, y \geq 0$.
(b) By using divergence theorem,
evaluate $\int_{s} \int\left(x^{3} d y d z+x^{2} y d z d x+x^{2} z d x d y\right)$ where S is closed surface bounded by the plane $z=0, z=b, x^{2}+y^{2}=a^{2}$.
Q. 6 State and prove Stoke's theorem.
Q. 6 Verify Stoke's theorem for $\bar{v}=y^{3} \bar{i}-x^{3} \bar{j}$ and surface S : the circular disk $x^{2}+y^{2} \leq 1, z=0$.

