(53 A & A-58-Eng) Seat No :________ No. of Printed Pages : 3

SARDAR PATEL UNIVERSITY

28th March, 2017, Tuesday

B.Sc - **SEM II , MATHEMATICS US02CMTH02**

(Matrix Theory & Differential Equations)

Time:2 Hours - 2 pm To 4 PM **Total Marks: 70**

1. Answer the following by selecting correct choice from the options : [10]

(1) A matrix $\begin{bmatrix} 7 & 0 \\ 0 & 5 \end{bmatrix}$ is ________ matrix.

(a) Scalar (b) Identity

(c) Diagonal (d) None

(2) If A and B are symmetric matrices, then ________

(a) AB is symmetric (b) BA is symmetric

(c) AB+BA is symmetric (d) AB is skew symmetric

(3) Characteristic roots of a skew Hermitian matrix are ________

(a) Real (b) Pure imaginary

(c) Zero (d) Zero or Pure imaginary

(4) If 3 is characteristic root of matrix A then _____

(a) $|I + 3A| = 0$ (b) $|I - 3A| = 0$

(c) $|A + 3I| = 0$ (d) $|A - 3I| = 0$

(5) A square matrix A is said to be an orthogonal matrix if ____________

(a) $AA^{-1} = I$ (b) $A = A^T$

(c) $AA^T = I$ (d) $AA^\theta = I$

(6) $\frac{1}{(D-1)^3} e^x =$ ________

(a) $\frac{x^3}{3!} e^x$ (b) $\frac{x^2}{3!} e^x$

(c) $\frac{x^3}{2!} e^x$ (d) $\frac{x^2}{3} e^x$

(7) The complementary function(C.F.) of $(D+3)^2 y = sinx$ is __________

(a) $c_1 + c_2 e^{3x}$ (b) $(c_1 + c_2 x)e^{3x}$

(c) $(c_1 + c_2 x)e^{-3x}$ (d) $(c_1 x + c_2 x^2)e^{-3x}$

(P.T.O.)

(8) The particular integral (P.I) of $(D^2+4)y = cos2x$ is ________

(a) $\frac{x}{8}cos2x$ (b) $\frac{x}{4}sin2x$

(c) $\frac{x}{2}cos2x$ (d) $-\frac{x}{4}sin2x$

(9) $\frac{1}{D^2}x^2 =$ ________

(a) $\frac{x^3}{3!}$ (b) $\frac{x^2}{12}$

(c) $\frac{x^4}{12}$ (d) $\frac{x^4}{3}$

(10) The solution of differential equation $(D^2+9)y = 0$ is ________

(a) $c_1cos3x + c_2sin9x$ (b) $c_1cos3x + c_2cos9x$

(c) $c_1cos3x + c_2sin3x$ (d) $e^{3x}(c_1cos3x + c_2sin3x)$

2. Answer any TEN of the following. [20]

1) Define : (i) Scalar Matrix (ii) Diagonal Matrix
2) Define singular matrix with an illustration.
3) Is matrix multiplication commutative? Justify your answer.
4) If $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ then find characteristic equation of A.
5) Define characteristic root and characteristic vector of Matrix.
6) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ then find characteristic root of A.
7) Find the value of $\frac{1}{D+1}(x^2+1)$
8) Find C.F. for the differential equation $(D^3-1)y = x^2$
9) Find the P.I. for the differential equation $(D^2-3D+2)y = sin2x$
10) Solve the differential equation $(D^4-1)y = 0$
11) Find the P.I. for the differential equation $(D^2-6D+5)y = e^{5x}$
12) Find C.F. for the differential equation $(D^2+9)y = cos3x$

3. (a) Prove that every square matrix can be expressed in one and only one way as the sum of a symmetric and skew symmetric matrix. [5]

(b) Prove that the product of matrices

$A = \begin{bmatrix} cos^2\theta & cos\theta sin\theta \\ cos\theta sin\theta & sin^2\theta \end{bmatrix}$, $B = \begin{bmatrix} cos^2\emptyset & cos\emptyset sin\emptyset \\ cos\emptyset sin\emptyset & sin^2\emptyset \end{bmatrix}$ is zero when θ and $\emptyset$ differ by an odd multiple of $\frac{\pi}{2}$ [5]

OR

3. (a) State and prove reversal law for the transpose of a product of matrices. **[5]**

(b) If $A_\alpha = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$ **then prove that** $(A_\alpha)^n = \begin{bmatrix} cosn\alpha & sinn\alpha \\ -sinn\alpha & cosn\alpha \end{bmatrix}$ **where** n **is positive integer.** **[5]**

4. (a) State and prove Cayley-Hamilton theorem. **[5]**

(b) Find the characteristic root and any one of the characteristic vector of the matrix **[5]**

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

OR

4. (a) Prove that the modulus of characteristic root of a unitary matrix is unity. **[5]**

(b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$ **[5]**

5. (a) In usual notations prove that $\frac{1}{f(D)} e^{mx} = \frac{1}{f(m)} e^{mx}$, $f(m) \neq 0$ **[5]**

(b) Solve the differential equation $(D^4 + D^3 + D^2 - D - 2)y = e^x + e^{-x}$ **[5]**

OR

5. (a) Solve the differential equation $(D^2 + a^2)y = secax$ **where** $a \in R$ **[5]**

(b) Solve the differential equation $(D^4 - 2D^2 + 1)y = e^{x/2}$ **[5]**

6. (a) In usual notations prove that $\frac{1}{\emptyset(D^2)} cosax = \frac{1}{\emptyset(-a^2)} cosax$, $\emptyset(-a^2) \neq 0$ **[5]**

(b) Solve the differential equation $(D^2 - 5D + 6)y = cos2x$ **[5]**

OR

6. (a) In usual notations prove that $\frac{1}{f(D)} e^{ax}V = e^{ax} \frac{1}{f(D+a)} V$ **, where** V **is function** x. **[5]**

(b) Solve the differential equation $(x^2D^2 + xD - 1)y = x^4$ **[5]**

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