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SARDAR PATEL UNIVERSITY
F.Y.B.Sc. (II SEM.) (CBCS) EXAMINATION

2011

Monday, 25th April

3.00 p.m. to 5.00 p.m.

MATHEMATICS

US02CMTH01: ANALYTICAL SOLID GEOMETRY

Total Marks: 70

Q:1 Indicate your choice of correct answer for each sub-questions in your answer-book by writing sub-question number and answer with latter a.b.c.d whichever is appropriate. **(10)**

(1) The centre of the sphere $x^2 + y^2 + z^2 + 2x + 2y - 2z - 6 = 0$ is a

- (a) (-1,1,1) (b) (-1,-1,1) (c) (-1,-1,-1) (d) (1,1,-1)

(2) The plane section of any sphere is

- (a) points (b) line (c) circle (d) sphere

(3) The radius of a sphere $x^2 + y^2 + z^2 - 2x - 4y - 4z = 7$ is

- (a) $\sqrt{2}$ (b) 2 (c) 3 (d) 4

(4) The equation $\frac{x^2}{9} - \frac{y^2}{4} - \frac{z^2}{25} = 1$ represents an

- (a) Elliptic hyperboloid of two sheets (b) Elliptic hyperboloid of one sheet (c) Elliptic cone (d) Elliptic paraboloid

(5) The xz-trace of the surface $\frac{x^2}{9} + \frac{y^2}{4} - \frac{z^2}{25} = 1$ is

- (a) Parabola (b) Hyperbola (c) Ellipse (d) Pair of lines

(6) In a cylindrical polar co-ordinate system the equation $z=2$ represents a plane

- (a) parallel to xy- plane (b) parallel to yz-plane

(c) parallel to zx-plane (d) none of these

(7) The vertex of a cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ is

- (a) (f,0,0) (b) (0,g,0) (c) (0,0,h) (d) none of these

(8) The condition that a cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ admits a set of three mutually perpendicular generators is

- (a) $ab+bc+ca=0$ (b) $ab+bc+ca=1$ (c) $a+b+c=0$ (d) $a+b+c=1$

(9) The vertex of reciprocal of a cone

$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ is

- (a) (0,0,0) (b) (f,g,h) (c) (-f,-g,-h) (d) (-F,-G,-H)

(10) Every plane sections of a right circular ~~cone~~ cylinder by a plane perpendicular to its axis is

- (a) Hyperbola (b) Rectangle (c) Circle (d) Ellipse

Q:2 Do as directed **(20)**

(1) Find the equation of a sphere with centre (2,-1,0) and passing through the point (1,-1,2).

(2) By using definition of a sphere derive the equation of a sphere with centre (α, β, γ) and radius 'a'.

* (9) Find the condition that the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ admits three mutually tangent planes.

(3) Find the equation of a tangent plane to a sphere

$$x^2 + y^2 + z^2 + 2x + 4y - 6z - 24 = 0 \text{ at a point } (1, 1, -2)$$

(4) By using standard form of quadric surface identify the surface given by $9x^2 - 4y^2 + 9z^2 = 0$.

(5) Find Jacobian for $u = x + 2y, v = 3x + y$.

(6) Plot the points $(2, 7\pi/6, \pi/6)$, and $(3, 40^\circ, 60^\circ)$ in R^3

(7) Find the equation of the cone with vertex at origin and direction cosine of whose generators satisfy the condition $l^2 + m^2 - n^2 = 0$.

(8) Find the equation of the cone with vertex at origin and which passes through the three co-ordinate axes.

* (10) Find the equation of right circular cone with vertex at origin and axis of cone be z-axis.

Q:3 (a) Two spheres given by $S_1 = x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ (6)

$$\text{and } S_2 = x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0. \text{ Prove that}$$

$\lambda S_1 + \lambda S_2 = 0$, where $\lambda \in \mathbb{R} - \{-1\}$, represents a family of spheres passing through the intersection of $S_1 \equiv 0$ & $S_2 \equiv 0$.

(b) Show that the spheres $x^2 + y^2 + z^2 = 64$ and (4)

$$x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0 \text{ touches each other.}$$

OR

Q:3 (c) Find the equation of the sphere which passing through the (5)

$$\text{circle } x^2 + y^2 + z^2 + 2x + 6y - 4z - 11 = 0;$$

$x^2 + y^2 + z^2 + 4x - 8y + 2z + 17 = 0$ and through the centre of one of the spheres.

(d) Find the equation of a tangent planes to the sphere (5)

$$x^2 + y^2 + z^2 + 2x - 2y - 2z - 1 = 0 \text{ which are parallel to the plane } 2x - y + 2z = 0$$

Q:4 (a) Identify and describe the surface given by (5)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, c > a, c > b.$$

(b) Show that $Ax^2 + By^2 + Cz^2 = D$ represents an elliptic hyperboloid (5)
of one sheet if one of the coefficient is negative and $D > 0$

OR

Q:4 (c) Describe the surface given by (1) $\rho = a$, a is positive constant (5)

(2) $\theta = 0$, in spherical polar co-ordinate system.

(d) By using standard form of quadratic surface identify the surface (5)

$$9x^2 + 4y^2 - 9z^2 - 18x - 8y - 18z = 32$$

Q:5 (a) Find the equation of a cone with vertex at (α, β, γ) and whose (6)

generators touches the sphere $x^2 + y^2 + z^2 = a^2, (a \neq 0)$

(b) Show that the cone whose vertex is at the origin and which (4)

passes through the curve of intersection of the sphere $x^2 + y^2 + z^2 = 3a^2$, and any plane at a distance "a" from the origin
Has mutually perpendicular generators.

OR

- Q:5 (c) The plane through OX and OY included an angle θ . Show that (5)
their line of intersection lies on the cone
 $z^2(x^2 + y^2) = x^2 y^2 \tan^2 \theta$

- (d) Find the equation of the cone with vertex at Origin and which (5)
passes through the curve $x^2 + y^2 = 4, z = 2$.

- Q:6 (a) Find the equation of the reciprocal cone of the cone (5)
 $2yz + 3zx + 4xy = 0$

- (b) Find the equation of the cylinder whose generator (5)
intersect the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, z = 0$,
and are parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

OR

- Q:6 (c) Find the condition that the line of intersection of the (5)
Plane $lx + my + nz = 0$ and a cone $ax^2 + by^2 + cz^2 = 0$,
 $fyz + gzx + hxy = 0$ should be coincident. cylinder

- (d) Find the equation of the right circular cylinder whose (5)
axis is $\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n}$ and
radius is r .

X=X=X

(3)